

UNDERSTANDING ASSET CORRELATIONS

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Abstract

We study low-frequency movements in US stock-bond correlations. Why were correlations highly positive during the 1970-1980's but then turned sharply negative around 2000? We document a strong inverse relation between stock-bond correlations and correlations between real economic growth and inflation. We find that inflation uncertainty drove stocks and bonds in the same direction pre-2000 but in opposite directions post-2000. Higher inflation uncertainty was bad for stock prices throughout 1965-2011 but its impact on nominal bond prices switched from negative to positive around 2000. This contradicts the existing view that higher inflation risk is always bad for bonds. This change coincided with a macroeconomic regime-shift in which inflation turned procyclical after having been countercyclical for several decades. Overall, we find that time-variation in the comovement of growth and inflation is important for understanding how inflation impacts asset prices. We explain our empirical findings using a long-run risk model featuring non-neutral inflation shocks and regime-shifts in the correlation between growth and inflation. Nominal bonds are risky in times of stagflation but provide insurance when inflation is procyclical. The model produces time-varying stock-bond correlations and provides a rational explanation for why dividend yields and nominal yields comove in data, often referred to as the Fed-model. This stands in sharp contrast to the usual explanation of inflation illusion.

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1 Introduction

The correlation between returns on US stocks and nominal government bonds has been close to zero on average in post-war data but has varied substantially over time as shown in Figure 1. From being highly positive in the 1970's and 1980's, correlations turned sharply negative in the early 2000's. We find that not only asset correlations but also several other relations between inflation, inflation uncertainty, and asset prices changed sign at the same time. This evidence suggest that a systematic change in the relation between asset prices and the macroeconomy occurred around 2000. We argue that a shift in the cyclical nature of inflation from a long-lived countercyclical state to a procyclical state is a plausible explanation for our empirical findings.

First, we provide new empirical evidence showing that movements in inflation uncertainty has caused returns on equity and nominal bonds to move together in some periods but in opposite directions in others. While the impact of inflation uncertainty on stock prices was negative throughout our sample period 1965-2011, we find that its impact on nominal bonds has varied over time. In fact, we find that the relation between inflation uncertainty and nominal bond prices switched sign from negative to positive around 2000. This is in contrast with existing findings that higher inflation uncertainty is uniformly bad news for bond prices.¹ Consistent with this shift, we find that inflation uncertainty predicted stock-bond correlations positively up to 2000 but negatively thereafter.

Second, we find that the negative or positive effect of inflation uncertainty on bond prices depends on whether inflation is negatively or positively correlated with real growth. We estimate a regime-switching model on growth, inflation, and asset returns and identify a regime-shift around 2000 in which asset correlations turned sharply negative and inflation turned procyclical after having been countercyclical for several decades. Our evidence suggests that inflation uncertainty lowers bond prices in times of countercyclical inflation but raises bond prices when growth and inflation are positively correlated. Overall, we find that time-variation in the comovement of growth and inflation is important for understanding how inflation impacts asset prices.

Third, we rationalize our findings in a long-run risk model incorporating non-neutral inflation

¹See for example Beechey (2008), Gürkaynak et al. (2008), and Wright (2011).

shocks and regime shifts in the correlation between real growth and inflation, allowing for both countercyclical and procyclical inflation regimes. An increase in inflation risk is always bad news for stock prices in the model but can be either positive or negative news for bond prices depending on the inflation regime. Nominal bonds are risky assets in times of stagflation but provide a hedge against bad times when inflation is procyclical. This mechanism allows the model to replicate the changing effects of inflation risk on asset prices and generates stock-bond correlations that switch sign.

Fourth, we contribute to the literature dealing with the comovement of nominal yields and dividend yields, sometimes referred to as the Fed-model.² Existing explanations rely on a behavioral concept in the form of inflation illusion in which investors basically discount real cash flows with nominal discount rates. An increase in inflation raises discount rates and lower stock prices, creating a positive correlation between dividend yields and nominal yields. Inflation illusion can account for the positive correlation between dividend yields and nominal yields during 1965-2000 but is not consistent with the large negative correlations observed post-2000 and during the 1930's. This raises questions about the validity of the inflation illusion explanation. Instead, we provide a fully rational explanation based on the changing effects of inflation uncertainty on stocks and bonds stemming from the time-varying relation between growth and inflation. Our theoretical model matches the large shifts in correlations observed in data.

While several papers have explored the role of time-varying macroeconomic volatility for asset prices (e.g., Bansal and Yaron, 2004, and Lettau et al., 2008), the focus of this paper is on the time-varying covariance between growth and inflation. To date, very little has been said about the changing joint dynamics of inflation and consumption growth and its effect on asset prices.

The focus of our study is to understand low-frequency movements in stock and bond returns as opposed to daily or even monthly fluctuations. Figure 2 plots the correlation between stock and bond returns against the correlation between real consumption growth and inflation for the period 1965-2011. All correlations are based on quarterly data and computed for non-overlapping five-year periods. The graph suggests a clear inverse relation between macro and asset correlations. Figures

²See for example Ritter and Warr (2002), Campbell and Vuolteenaho (2004), Cohen et al. (2005), and Bekaert and Engstrom (2010).

3 and 4 show that the same pattern holds for other measures of economic activity such as real GDP and industrial production. The plots suggest that inflation has become procyclical throughout the 2000's in contrast to the stagflationary period of the 1970's and 1980's. At the same time, the correlation between stock and bond returns switched sign in the early 2000's and turned sharply negative.³

Figure 5 shows that the same pattern holds for the relation between nominal yields and dividend yields, sometimes referred to as the Fed-model in the literature. From being highly positively correlated throughout most of the sample period, the correlation turned sharply negative in the early 2000's. Since nominal yields are closely linked to the level of inflation, this suggests that the relation between equity valuations and inflation has changed considerably over time and seems to coincide with changes in the cyclical nature of inflation.

So is there any plausible economic link between the cyclical nature of inflation and the empirical observations described above? Yes. Asset-pricing theory suggests that nominal bonds are risky when inflation is countercyclical since their returns then are procyclical. In contrast, nominal bonds provide a hedge against bad times when inflation is procyclical. An increase in inflation risk should therefore raise bond risk premia in the first case while it should lower risk premia in the second case. Expected returns on equity should load positively on inflation risk if inflation is bad for growth and leads to poor equity returns, producing procyclical stock returns. The same also holds if inflation is positively associated with economic growth and stock returns since high inflation then coincides with low marginal utility of investors coupled with high returns, again generating procyclical returns. This suggests that risk premiums on equity should consistently load positive on inflation risk while bond risk premia can load negatively or positively depending on the cyclical state of inflation.

This line of reasoning suggests that inflation uncertainty per se, for example measured by inflation volatility or dispersion of inflation forecasts, does not determine whether nominal bonds are risky assets or not. Instead, it depends on whether inflation is pro or countercyclical. Inflation risk merely acts as a multiplier.

³The purpose of these figures is to provide motivating examples of joint changes in macro and asset correlations. We conduct more formal tests in Section 3 of the paper.

We rationalize our empirical observations using a long-run risk model which makes use of recursive preferences, persistent macroeconomic shocks, and time-varying macroeconomic volatility. The setup follows Bansal and Yaron (2004), but in contrast to existing long-run risk models, the focus is on persistent long-run inflation shocks rather than shocks to consumption growth. The key mechanism is that inflation has real effects and affects the real pricing kernel directly. Inflation therefore has an impact on real asset prices and on both equity and bond risk premia. As a result, expected returns on equity and bonds vary with inflation volatility. The real impact of inflation makes expected inflation a state variable for price-dividend ratios and real bonds, creating an immediate link between inflation and real asset prices. We restrict the volatility of consumption growth to be constant in order to focus solely on inflation risk.⁴

We also contribute to the long-run risk literature by incorporating a regime-switching mechanism in the model that allows for both countercyclical and procyclical inflation regimes. This feature makes the market price of long-run inflation risk dependent on the inflation regime and it can therefore switch sign. When inflation is bad news for the economy, as in periods of stagflation, both stocks and bonds are risky assets with respect to inflation risk which makes their risk premiums comove positively. When inflation is procyclical, nominal bonds provide a hedge against inflation risk while stocks still are risky. In this case, risk premiums on bonds and equity comove negatively. Hence, risk premiums on equity and bonds both vary with inflation risk in the model but the loading of bond risk premia on inflation risk can be either positive or negative depending on the cyclical nature of inflation. This mechanism allows the model to match the switching stock-bond correlations observed in data, the changing correlation between dividend yields and nominal yields, and the other shifting relations between inflation and asset prices that we document.

We do not claim that changes in inflation uncertainty is the only driver of stock and bond correlations. There are of course other potential candidates.⁵ However, we believe it is an important factor since we find that it is capable of producing both positive and negative correlations and therefore potentially explain the switching behavior in data.

⁴Allowing for time-varying consumption growth volatility does not change the main results. It is kept constant solely for parsimonious reasons.

⁵For example, Baele et al. (2010) find that liquidity factors play an important role.

Identifying the structural source behind changes in the cyclical behavior of inflation is an important question. One possible underlying mechanism is the nature of demand and supply shocks affecting the economy. A large body of macroeconomic literature deals with identifying such shocks and linking it to the cyclical behavior of output, employment, and prices.⁶ It is also conceivable that time variation in the stance of monetary policy plays an important role. However, a careful analysis along these lines is outside the scope of this paper. We focus instead on the direct consequences for asset prices stemming from changing inflation regimes.

Our article proceeds as follows. Section 2 provides a discussion of related literature. Section 3 describes our data and provides new empirical evidence of the relations between stock and bond returns and between inflation and asset prices in general. Section 4 rationalizes our empirical findings using a long-run risk model that incorporates non-neutral inflation shocks and a regime-switching mechanism allowing for both counter and procyclical inflation regimes. Section 5 describes the calibration of the theoretical model and shows that the model is capable of matching important asset-pricing moments while explaining the changing relations between inflation, equity, and bonds. Section 6 describes in detail the asset-pricing implications of the model for bonds, equity, and asset correlations. Section 7 concludes.

2 Related Literature

The relation between stock and bond returns has received great academic interest and is of central importance in for example asset allocation decisions. Early contributions focus on the unconditional correlation. Shiller and Beltratti (1992) fail to match the observed comovement using a present-value model. Campbell and Ammer (1993) decompose the variance of stock and bond returns and find offsetting effects from changes in real interest rates, excess returns, and expected inflation.

Recently, the focus has shifted towards understanding conditional correlations. Connolly et al. (2005) document a negative relation between stock market uncertainty and the stock-bond return correlation. Baele et al. (2010) find that macro factors have limited success in explaining the time-

⁶An incomplete list of relevant papers are: Mills (1927), Blanchard and Quah (1989), Kydland and Prescott (1990), Cooley and Ohanian (1991), and Stadler (1994)

varying return correlation and argue that liquidity factors play an important role. Campbell et al. (2010) specify and estimate a reduced form model in which one of the latent state variables is the covariance between inflation and the real economy. They link this state variable to the covariance of stock and bond returns and describe how the covariance term impacts bond risk premia and the shape of the yield curve. David and Veronesi (2009) explore the role of learning about inflation and real earnings for the second moments of stock and bond returns. Investors in their model are uncertain about the current state of the economy and any perceived deviations from the “normal” state, either to a “good” or “bad” state, increases uncertainty and asset return volatilities. The valuation of bonds and stocks, however, react to the directional change in economic states. This create a V-shaped relation between return volatilities and asset valuations.

In contrast to these papers, we address asset correlations and the relation between inflation and asset prices using a consumption-based equilibrium model. This ties asset prices directly to fundamental macro factors such as consumption growth and inflation. We also explicitly measure the changing covariance between consumption growth and inflation in data and show that a changing cyclical behavior of inflation has important implications for asset prices. We also show that the relation between inflation uncertainty and bond prices depends on the current inflation regime.

The correlation between US dividend yields and nominal interest rates has been positive throughout most of the post-war period and is commonly referred to as the Fed-model. This correlation has been considered puzzling since it implies that nominal interest rates, driven mainly by inflation, are correlated with a real variable. As is well known, dividend yields are negatively related to real cash-flow growth and positively related to expected returns. So in order to explain the positive correlation, inflation must be negatively related to real dividend growth rates and/or positively associated with expected returns.

Virtually all papers in the so-called Fed-model literature rule out a rational explanation for the link between inflation and dividend yields. Instead, inflation illusion has been advocated as the likely explanation, originally put forward by Modigliani and Cohn (1979).⁷ Inflation illusion suggests that investors are irrational and fail to adjust expected nominal dividend growth rates

⁷Among the papers that argue in favor of inflation illusion are Ritter and Warr (2002), Campbell and Vuolteenaho (2004), and Cohen et al. (2005).

with changes in expected inflation but fully adjust nominal discount rates. Alternatively, one can view it as investors are discounting real cash flows with nominal interest rates, creating a positive correlation between inflation and dividend yields. What is rarely mentioned in the literature is that the correlation between dividend yields and nominal yields in fact has been highly negative throughout the 2000's. This observation questions inflation illusion as a viable explanation since it can only explain a positive comovement.

In a recent empirical paper, Bekaert and Engstrom (2010) argue that rational mechanisms are at work and ascribe the positive correlation to the large incidence of stagflation in US data. They show that the correlation between dividend yields and bond yields is mainly driven by a correlation between expected inflation and the equity risk premium as periods of high expected inflation are associated with periods of high risk aversion and high economic uncertainty. This piece of evidence suggests that any rational equilibrium model that would like to explain the Fed-model must contain a link between inflation and the equity risk premium. The model in this paper contains exactly that.

This paper is also related to the vast literature on whether stocks are a good or bad inflation hedge. Several articles have established a negative relation between inflation and stock returns. Fama and Schwert (1977) document that common stock returns are inversely related to expected inflation. Fama (1981) argues that a rise in expected output is expected to lead to a drop in inflation. Since stock prices rise in anticipation of higher output, a negative correlation between stock returns and inflation arises. Fama's proxy hypothesis is distinct from this paper since we stress the link between inflation and risk premiums rather than inflation and cash flows. Bekaert and Wang (2010) study the relation between inflation and stock returns across a large number of countries and find evidence of predominantly negative inflation betas obtained by regressing nominal stock returns onto inflation.

We also build on the large literature on regime-switching models in equity and bond markets. For example, Cecchetti et al. (1990), Whitelaw (2000), Ang and Chen (2002), Bibkov and Chernov (2008), Lettau et al. (2008), Constantinides and Ghosh (2011), and Bonomo et al. (2011) study regime-switching models in connection to equity markets while Hamilton (1988), Garcia and Perron

(1996), Gray (1996), Evans (1998), Ang and Bekaert (2002), Bansal and Zhou (2002), Bansal et al. (2004), Dai et al. (2007), and Ang et al. (2008) all study bond markets.

This paper builds more generally on the literature of pricing stocks and bonds in equilibrium. Early contributions include Cox et al. (1985), Mehra and Prescott (1985), Campbell (1986), and Dunn and Singleton (1986). The recursive preferences of Epstein and Zin (1989) and Weil (1989) have been used extensively in the asset-pricing literature (e.g., Campbell, 1993, 1996, 1999, Duffie et al., 1997, and Restoy and Weil, 1998). Bansal and Yaron (2004) show that recursive preferences in conjunction with a time-varying first and second moment of consumption growth can explain the level of the equity risk premium and its variation over time. Piazzesi and Schneider (2006) make use of recursive preferences and show that the nominal yield curve slopes up if inflation is bad news for future consumption growth and explore the role of learning about inflation for interest rates. Hasseltoft (2011) estimates the long-run risk model using a simulation estimator and shows that it does well in explaining key features of both equity and bond markets. Bansal and Shaliastovich (2010) show that the long-run risk model can explain violations of both the expectation hypothesis in bond markets and the uncovered interest rate parity in currency markets.⁸

3 Empirical Analysis

3.1 Data

Quarterly aggregate US consumption data for the period 1965:1-2011:4 and annual data for the period 1930-2011 on nondurables and services is collected from the Bureau of Economic Analysis. Real consumption growth and inflation are computed as in Piazzesi and Schneider (2006) using the price index that corresponds to the consumption data. Value-weighted market returns (NYSE/AMEX) are retrieved from CRSP. Nominal interest rates and bond returns are collected from the Fama-Bliss files in CRSP. Price-dividend ratios are formed by imputing dividends from monthly CRSP returns that includes and excludes dividends (e.g., Bansal et al., 2005). Quarterly dividends D_t are formed by summing monthly dividends. Due to the strong seasonality of dividend

⁸Other papers that model the term structure of interest rates in a long-run risk setting include Eraker (2008), Wu (2008), and Doh (2010).

payments, we use a four-quarter moving average of dividend payments, $\bar{D}_t = \frac{D_t + D_{t-1} + D_{t-2} + D_{t-3}}{4}$. Real dividend growth rates are found by taking the log first difference of \bar{D}_t and deflating using the constructed inflation series. Data on industrial production and real GDP are obtained from the St. Louis FRED database and from GlobalFinancialData respectively.

3.2 Empirical Evidence

The introduction of the paper provided motivating examples for the inverse relation between asset and macro correlations found in data. Table 1 reports the actual correlation coefficients between growth, inflation, and asset returns. Motivated by the preceding figures, we choose to break the sample in two parts, prior to and after 2000.⁹ The table shows that inflation and growth were negatively correlated prior to 2000, -0.42 , but comoved positively thereafter, 0.30 . The correlations between stock and bond returns also underwent a significant change, from 0.28 pre 2000 to -0.63 thereafter. Interestingly, the correlation between inflation and stock returns also changed from -0.20 to 0.27 after 2000. Overall, this suggest that several relations switched around year 2000; inflation turned procyclical, stock-bond correlations turned sharply negative and equity switched to act as good inflation hedge. We argue that these observations are all interlinked.

In Table 2, we test formally whether the difference in unconditional correlation coefficients pre and post 2000 are statistically significant using a Jennrich (1970) test. We consider three different correlation matrices; macro variables and asset returns, only macro variables, and only asset returns. The table shows that we can clearly reject the null hypothesis of equal correlation coefficients across subsamples for all specifications. It has been argued in the literature that simply splitting the sample ex-post is not a perfectly reliable method of identifying the point of a structural break as it might lead to a selection bias (e.g. Boyer et. al, 1999, and Chesnay and Jondeau, 2001). We therefore, late in this section, estimate a regime-switching model that explicitly allows us to formally identify a breakpoint between countercyclical and procyclical inflation regimes.

To further validate the inverse relation between macro and asset correlations, we take a long-term perspective by considering annual consumption growth and inflation starting in 1930 in Figure

⁹We estimate a more formal breakpoint later in this section.

6. Visual inspection suggests that the comovement between inflation and growth has varied considerably. In particular, the 1930's experienced a strong positive comovement as The Great Depression was associated with low growth coupled with deflation. The positive correlation was further exacerbated by the strong rebound in growth and inflation starting in 1933. In contrast, the US economy underwent a significant stagflationary period in the 1970's and early 1980's. Furthermore, the figure shows that consumption growth and inflation started to comove positively again around the year 2000.

Figure 7 computes 10-year correlations between growth and inflation and between dividend yields and inflation for the period 1930-2011.¹⁰ First, the graph quantifies the considerable variation in the cyclical nature of inflation. From a correlation of 0.80 in the 1930's, correlations reached -0.80 during the 1970's and were then close to 0.60 in the 2000's. Second, the graph highlights clearly that the inverse relation between macro correlations and correlations between dividend yields and inflation is robust when extending the time period.

We also provide new evidence that not only asset correlations switched sign in the early 2000's but also several other relations between inflation and asset prices. Figure 8 shows that the relation between the level of expected inflation and price-dividend ratios switched sharply from negative to positive.¹¹ This is in line with our earlier finding that stock returns were positively related to inflation after 2000 and in fact provided a hedge against inflation. This stands in contrast to existing findings that stocks usually are poor inflation hedges, at least in the short term. We argue in this paper that this switch stems from the changing cyclical nature of inflation. Interestingly, the same figure shows that inflation risk has been consistently negatively related to price-dividend ratios throughout the entire sample period. Hence, stock prices seem to respond differently to changes in the level of inflation and changes in inflation risk.¹² The model we present later provides a rational explanation for this phenomenon.

¹⁰Since interest rates were not market determined prior to the Treasury-Fed accord in 1951, we choose to focus on the relation between equity and inflation as opposed to using bond returns.

¹¹Expected inflation is measured as the fitted value from projecting quarterly inflation onto lagged growth, inflation, and yield spread.

¹²Inflation risk is here measured as the dispersion of inflation forecasts based on the GDP price deflator. We show in the appendix that the general results do not change if inflation risk instead is measured as the dispersion of inflation forecasts based on CPI or measured as the conditional volatility of inflation estimated from a GARCH(1,1) model.

Table 3 reports results from regressing log price-dividend ratios onto expected inflation and inflation risk for the full sample and for the two subsamples. First, the two variables explain a large part of the variation in price-dividend ratios with R^2 s around 50%. Second, the regression coefficient for expected inflation switches sign to positive in the procyclical state, albeit not statistically significant. Third, inflation risk is consistently negatively related to price-dividend ratios with a high statistical significance. Hence, the table suggests that while the relation between equity-valuation ratios and the level of inflation may change sign, an increase in inflation risk always depresses equity prices.

Conventional wisdom suggests that an increase in inflation risk should raise nominal yields and therefore affect bond returns negatively. Figure 9 shows that this is not always true. In fact, inflation risk and nominal interest rates have been highly *negatively* correlated throughout the last 10 years. We believe this is a novel finding and raises the question of what the underlying mechanism is. The answer is, as discussed above, that the riskiness of nominal bonds depends on whether inflation is counter or procyclical. Since nominal bonds are risky assets in periods of countercyclical inflation, their returns will suffer in periods of high inflation risk. Conversely, nominal bonds provide a hedge against bad times when inflation is procyclical. This makes their returns positively related to inflation risk.

Table 4 supports these findings by reporting results from regressing the 5-year nominal interest rate onto expected inflation and inflation risk. We find that the regression coefficient for inflation risk switches sign to negative in the procyclical state and is highly statistical significant. This result does not depend on the maturity of the bond but holds across the entire yield curve.

Our findings so far suggest that movements in inflation risk drive stock and bond returns in the same direction if inflation is countercyclical but in opposite directions if inflation is procyclical. This means that inflation risk should predict stock-bond correlations either positively or negatively depending on the current inflation regime. We test this by predicting the quarterly covariance of stock and bond returns using lagged inflation risk. Table 5 reports the results. As predicted, the sign of the regression coefficient switches sign to negative for the procyclical period. While it has been documented elsewhere that inflation volatility predicts the stock-bond covariance positively

(e.g. Viceira, 2010), we are not aware of any paper showing that this relation can switch sign.

In order to more formally analyze the relation between macro and asset correlations and to estimate a formal structural break point, we estimate a two-state Markov-switching (MS) model in which quarterly consumption growth, inflation, excess stock returns, and excess bond returns are assumed to follow a one-lag vector autoregression:

$$Y_{t+1} = \mu(s_{t+1}) + \beta(s_{t+1})Y_t + \epsilon_{t+1}, \quad (1)$$

where $Y_{t+1} = [\Delta c_{t+1}, \pi_{t+1}, r_{s,t+1}, r_{b,t+1}]'$, $\epsilon_{t+1} \sim N(0, \Omega(s_{t+1}))$ and where the regime s_{t+1} is presumed to follow a two-state Markov chain with transition probabilities $p_{ij} = P(s_{t+1} = j | s_t = i)$. The probability of ending up in tomorrow's regime $s_{t+1} = (0, 1)$ given today's regime $s_t = (0, 1)$ is governed by the transition probability matrix of a Markov chain:

$$P = \begin{bmatrix} p_{00} & p_{10} \\ p_{01} & p_{11} \end{bmatrix},$$

where $\sum_{j=0}^1 p_{ij} = 1$, and where $0 < p_{ij} < 1$.

Stock returns refer to the NYSE/AMEX portfolio available from CRSP and bond returns refer to a 5-year US Treasury bond also obtained from CRSP. Short rates used to compute excess returns come from the Fama files in CRSP. All variables are observed quarterly. The companion matrix β and the variance-covariance matrix Σ are assumed to follow the same Markov chain process yielding two possible states in total.¹³

The estimation results are reported in Table 6. There are three key takeaways from this table. First, the effect of inflation on future growth (element (1,2) in β) and the covariance between growth and inflation shocks (element (1,2) in Ω) both switch from negative to positive in the second state. Although estimates for the second state are subject to rather large standard errors, the numbers are consistent with the fact that the correlation between growth and inflation turned

¹³The setup follows Hamilton (1989,1994) among others. We have estimated various MS-VAR specifications allowing for a larger number of states, more lags in the VAR etc.. We have also elaborated with time-varying transition probabilities. They all yield similar qualitative conclusions. Additional estimation results are available upon request.

positive in the second state. Second, the covariance between shocks to stock and bond returns changed even more dramatically from positive to negative in the second state (element (3,4) in Ω), where both covariance terms are statistically significant. This is consistent with our earlier evidence that correlations between stocks and bonds turned sharply negative in the early 2000's. Third, the covariance between shocks to inflation and stock returns (element (2,3) in Ω) switched from negative to positive in the second state, consistent with our earlier evidence that stocks seem to have provided an inflation hedge throughout the 2000's.

Table 7 reports the unconditional correlation coefficients between consumption growth, inflation, stock, and bond returns implied by our estimated Markov-switching model. Correlations for pre- and post-2000 (countercyclical versus procyclical period) are reported and are shown to be close to the actual correlations based on data that we earlier reported in Table 1. Table 8 conducts Jennrich (1970) tests of the implied correlation coefficients and shows that the null hypothesis of equal correlation matrices across the two sub-periods can be rejected with a high statistical significance.

The transition probabilities, p_{11} and p_{22} , are estimated to 0.977 and 0.939. This implies unconditional probabilities of 0.73 and 0.27 for being in the counter versus procyclical state. The estimated probabilities also imply a longer expected duration for the countercyclical state, 43 quarters, versus 16 quarters for the procyclical inflation state.¹⁴

Next, we test the null hypothesis that conditional correlations are constant across regimes using a Likelihood Ratio test. Specifically, we test the restriction $\rho(s_t = 0) = \rho(s_t = 1)$ for each of our six possible pairs of variables. Table 9 reports the test results and shows that we can clearly reject the null of constant conditional correlations across regimes.

Using our estimation results we compute the implied probability of being in the procyclical regime at each time t . We consider both filtered regime probabilities that uses information up to time t and smoothed regime probabilities that uses information from the entire sample. The probabilities are plotted in Figure 10. We find that a large part of the sample period is dominated by the countercyclical state featuring a negative correlation between growth and inflation. We find

¹⁴Considering two states, i and j , the unconditional probability of being in state j is computed as $\frac{1-p_{ii}}{2-p_{jj}-p_{ii}}$. The expected duration of state i is computed as $\frac{1}{1-p_{ii}}$.

a temporary jump to the procyclical state in the late 1980's and then a more long-lasting move to a procyclical state in the early 2000's, encompassing virtually the entire 2000's. Large opposite shocks to growth and inflation in the fourth quarter of 2007 explains the large drop in smoothed probabilities at the end of the sample. However, a large deflationary shock together with a sharp drop in growth in the fourth quarter of 2008 explains the reversal in probabilities. We find that the smoothed probability exceeds 0.50 in the second quarter of 2001 wherefore we treat this as the formally estimated breaking point between the two inflation regimes.

From the estimated regime-switching model we then compute a time-series of conditional correlations between growth and inflation and between stock and bond returns and plot them in Figure 11. First, the relative movements of the two lines suggest a clear inverse relation, particularly in the late 1980's, the early 2000's, and recently around the financial crisis. Second, the absolute movements of the correlations are also consistent with our earlier evidence that macro and asset correlations underwent a significant shift in the early 2000's.

Overall, we have shown that a number of relations between inflation and asset prices have switched sign over time. Our evidence suggest that the switch in signs all occurred at the same time, namely around 2000. We have shown that these observations line up with a contemporaneous switch in the correlation between growth and inflation, from negative to positive. We argue in this paper that incorporating time-varying correlations between inflation and the real side of the economy helps explain these puzzling relations between inflation and asset prices.

4 A Long-Run Risk Model with Switching Inflation Regimes

This section presents a consumption-based model with a representative agent that is able to provide a rational explanation for our empirical findings while at the same time matching a range of important macro and asset-price moments.

The model builds on the so-called long-run risk literature which relies on Epstein and Zin (1989) and Weil (1989) recursive preferences, persistent macro shocks, and time-varying macroeconomic volatility. The original long-run risk model of Bansal and Yaron (2004) relies on persistent shocks to expected consumption growth which together with the preference specification produces a sizeable

equity risk premia. Inflation plays no role in that model.

In contrast, this paper focusses on long-run shocks to inflation and their effect on consumption growth and asset prices. The model contains two additional key features compared to the standard long-run risk model: First, long-run inflation shocks are non-neutral and impact future real economic growth. This allows inflation to have a direct impact on the real pricing kernel and therefore on both equity and bond risk premia. As a result, expected returns on equity and bonds vary with the conditional variance of inflation. Second, we incorporate a Markov-switching regime mechanism that allows the relation between real growth and inflation to switch sign. This allows the model to produce both a counter and procyclical inflation regime. We find this ingredient to be crucial for explaining the switching relations between inflation and asset prices in general and between stock and bond returns in particular. In general, we find that these two main distinctions from the original long-run risk model open up a range of novel asset-pricing implications.

4.1 Macro Dynamics

Let Δc_{t+1} , π_{t+1} , Δd_{t+1} , and $\sigma_{\pi,t+1}^2$ denote the logarithmic consumption growth, inflation, dividend growth, and the conditional variance of inflation respectively. Let μ_c , μ_π , and μ_d denote the unconditional means and let x_c and x_π denote the time-varying part of the conditional means. We consider an economy subject to regime-shifts between two possible states. Tomorrow's regime is denoted $s_{t+1} = (0, 1)$ and the probability of ending up in tomorrow's regime given today's regime $s_t = (0, 1)$ is governed by the transition probability matrix of a Markov chain:

$$P = \begin{bmatrix} p_{00} & p_{10} \\ p_{01} & p_{11} \end{bmatrix},$$

where $P(s_{t+1} = j | s_t = i) = p_{ij}$, $\sum_{j=0}^1 p_{ij} = 1$, and where $0 < p_{ij} < 1$. We assume that agents are able to observe the current regime.

We assume the following macro dynamics:

$$\Delta c_{t+1} = \mu_c(s_{t+1}) + x_{c,t} + \sigma_c \eta_{c,t+1}, \quad (2)$$

$$\pi_{t+1} = \mu_\pi(s_{t+1}) + x_{\pi,t} + \sigma_{\pi,t} \eta_{\pi,t+1}, \quad (3)$$

$$\Delta d_{t+1} = \mu_d + \phi x_{c,t} + \varphi \sigma_c \eta_{d,t+1}, \quad (4)$$

$$\sigma_{\pi,t+1}^2 = \sigma_\pi^2 + v_\pi (\sigma_{\pi,t}^2 - \sigma_\pi^2) + \sigma_\nu w_{t+1}, \quad (5)$$

$$\begin{pmatrix} x_{c,t+1} \\ x_{\pi,t+1} \end{pmatrix} = \begin{pmatrix} \beta_1(s_{t+1}) & \beta_2(s_{t+1}) \\ 0 & \beta_4(s_{t+1}) \end{pmatrix} \begin{pmatrix} x_{c,t} \\ x_{\pi,t} \end{pmatrix} + \begin{pmatrix} \delta_1(s_{t+1}) & \delta_2(s_{t+1}) \\ \delta_3(s_{t+1}) & \delta_4(s_{t+1}) \end{pmatrix} \begin{pmatrix} \sigma_c \varepsilon_{c,t+1} \\ \sigma_{\pi,t} \varepsilon_{\pi,t+1} \end{pmatrix}, \quad (6)$$

where all shocks are uncorrelated, i.i.d. normally distributed with a mean of zero and a variance of one. The β and δ matrices plus μ_c and μ_π depend on tomorrow's regime $s_{t+1} = (0, 1)$. We set element (2,1) in β equal to zero since it does not affect our qualitative findings. Doing so also helps us to provide intuitive analytical model solutions. For parsimonious reasons, we keep the volatility parameters and the dividend growth parameters constant across regimes. We do this since our main focus is on the interaction between growth and inflation and the corresponding effects on asset prices.

The presence of regime-shifts, inflation, and inflation volatility are new compared to the specification in Bansal and Yaron (2004). While the process for dividend growth is identical to the original long-run risk model, the specification for realized and expected growth in Bansal and Yaron (2004) using our notation would be: $\Delta c_{t+1} = \mu_c + x_{c,t} + \sigma_c \eta_{c,t+1}$ and $x_{c,t+1} = \beta_1 x_{c,t} + \delta_1 \sigma_c \varepsilon_{c,t+1}$ respectively.

Time-varying volatility of inflation $\sigma_{\pi,t+1}^2$ is the only source of time-variation in second moments and gives rise to time-varying risk premiums as shown in Section 6. Note that the typical long-run risk specification relies on time-varying volatility of consumption growth as opposed to inflation. We have restricted time-variation in second moments to inflation since our focus is on the impact of inflation on asset prices. Hence, the only channel of macroeconomic uncertainty is inflation uncertainty. The notion of heteroscedasticity in inflation is a well established empirical fact; early

contributions include Engle (1982) and Bollerslev (1986). The state variables of the model are consequently $x_{c,t}$, $x_{\pi,t}$, and $\sigma_{\pi,t}^2$.¹⁵

The conditional means of consumption growth and inflation are interdependent through β_2 which means that expected future growth depends on today's expected inflation. β_2 is a key parameter and represents a distinguishing feature with respect to other long-run risk specifications. More specifically, β_2 allows real asset prices and valuation ratios such as real bonds and price-dividend ratios, to be functions of expected inflation since the conditional mean of $x_{c,t+1}$ depends on $x_{\pi,t}$. β_2 creates a direct link between expected inflation and the real pricing kernel which means that inflation affects risk premiums in the economy. Importantly, β_2 allows inflation to affect not only bond risk premia but also equity risk premia. Since β_2 is subject to regime shifts, it can take on both positive and negative values. This shift in sign is important for matching our empirical findings that the growth-inflation correlation and the loading of price-dividend ratios onto expected inflation changes sign over time.

This specification is the most parsimonious setup that is able to match both key macro and asset-price moments and the switching behavior between inflation and asset prices. One could of course relax several of the restrictions. For example, we could allow for a non-zero β_3 , i.e. an interaction between $x_{c,t}$ and $x_{\pi,t+1}$. We could allow dividend growth parameters and volatility parameters to also change across regimes. We could allow the entire variance-covariance matrix of growth and inflation to vary with time. Hasseltoft (2009) contains some of these relaxations. Overall, giving more flexibility to the model does not change our main qualitative results.

4.2 Investor Preferences

The representative agent in the economy has Epstein and Zin (1989) and Weil (1989) recursive preferences:

$$U_t = \left\{ (1 - \delta)C_t^{\frac{1-\gamma}{\theta}} + \delta(E_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}, \quad (7)$$

¹⁵It is straightforward to also allow for time-varying consumption growth volatility. Doing so does not change the qualitative results of the paper.

where $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, $\gamma \geq 0$ denotes the risk aversion coefficient and $\psi \geq 0$ the elasticity of intertemporal substitution (EIS). The discount factor is represented by δ . This preference specification allows time preferences to be separated from risk preferences. This stands in contrast to time-separable expected utility in which the desire to smooth consumption over states and over time are interlinked. The agent prefers early (late) resolution of risk when the risk aversion is larger (smaller) than the reciprocal of the EIS. A preference for early resolution and an EIS above one imply that $\theta < 1$. This specification nests the time-separable power utility model for $\gamma = \frac{1}{\psi}$ (i.e., $\theta = 1$).

The agent is subject to the following budget constraint $W_{t+1} = R_{c,t+1}(W_t - C_t)$ where the agent's total wealth is denoted W_t , $W_t - C_t$ is the amount of wealth invested in asset markets and $R_{c,t+1}$ denotes the gross return on the agents total wealth portfolio. This asset delivers aggregate consumption as its dividends each period.

Following Epstein and Zin (1989) plus acknowledging the presence of regime-shifts, the logarithm of the stochastic discount factor (SDF) can be written as:

$$m_{t+1}(s_{t+1}) = \theta \ln(\delta) - \frac{\theta}{\psi} \Delta c_{t+1} - (1 - \theta) r_{c,t+1}(s_{t+1}), \quad (8)$$

where $\ln R_{c,t+1} = r_{c,t+1}$. Note that the SDF depends on both consumption growth and on the return from the total wealth portfolio. Recall that $\theta = 1$ under power utility, which brings us back to the standard time-separable SDF.

4.3 Solving the model

Returns on the aggregate wealth portfolio and the market portfolio are approximated as in Campbell and Shiller (1988):

$$r_{c,t+1}(s_{t+1}) = k_{c,0} + k_{c,1} p c_{t+1}(s_{t+1}) - p c_t(s_t) + \Delta c_{t+1}, \quad (9)$$

$$r_{m,t+1}(s_{t+1}) = k_{d,0} + k_{d,1} p d_{t+1}(s_{t+1}) - p d_t(s_t) + \Delta d_{t+1}, \quad (10)$$

where pc_t and pd_t denote the log price-consumption ratio and the log price-dividend ratio.¹⁶ The constants k_c and k_d are functions of the average level of pc_t and pd_t , denoted \bar{pc} and \bar{pd} .¹⁷

4.4 Solving for Equity

All asset prices and valuation ratios are conjectured to be functions of the time-varying conditional means of consumption growth and inflation plus the time-varying conditional variance of inflation. Starting with the log price-consumption ratio, it is conjectured to be a linear function of the state variables as follows:

$$pc_t(s_t) = A_{c,0}(s_t) + A_{c,1}(s_t)x_{c,t} + A_{c,2}(s_t)x_{\pi,t} + A_{c,3}(s_t)\sigma_{\pi,t}^2. \quad (11)$$

The regime dependence of the coefficients plus the existence of $A_{c,2}$ and $A_{c,3}$ are new compared to Bansal and Yaron (2004). $A_{c,2}(s_t)$ and $A_{c,3}(s_t)$ arise from the fact that inflation has a direct impact on real economic growth through $\beta_2(s_t)$ and $\delta_2(s_t)$. In order to solve for the A-coefficients we make use of the macro dynamics, the law of iterated expectations, and of the Euler equation for the consumption asset. Appendix A.1 describes in detail how we solve the model and contains the analytical expressions for the A-coefficients. Note that our expression for $A_{c,1}(s_t)$ collapses to the same expression as in Bansal and Yaron (2004) should we restrict the model to a single regime.

Since the analytical expressions for the A-coefficients are rather complicated, we choose to intuitively describe what drives the values of the A-coefficients rather than mechanically discuss them. $A_{c,1}(s_t)$ is positive whenever the elasticity of intertemporal substitution (EIS) is greater than one, implying a positive relation between expected growth and asset prices in both regimes. $A_{c,2}(s_t)$ represents the loading of the price-consumption ratio on expected inflation. Its sign depends on mainly on $\beta_2(s_t)$ and on the EIS. Recall that $\beta_2(s_t)$ governs how expected inflation affects future growth. For $\beta_2(s_t) < 0$, high inflation signals negative future growth and will therefore depress the price-consumption ratio, i.e., $A_{c,2}(s_t) < 0$. The opposite holds when $\beta_2(s_t) > 0$ as high inflation

¹⁶Bansal et al. (2007a) show that the approximate analytical solutions for the returns are close to the numerical solutions and deliver similar model implications.

¹⁷Specifically, the constants are $k_{c,1} = \frac{\exp(\bar{pc})}{1+\exp(\bar{pc})}$ and $k_{c,0} = \ln(1 + \exp(\bar{pc})) - k_{c,1}\bar{pc}$ and similarly for the k_d coefficients.

then signals positive future growth and therefore higher price-consumption ratios, $A_{c,2}(s_t) > 0$. This only holds when the EIS exceeds one meaning that the intertemporal substitution effect dominates the wealth effect. In the case of expected utility ($\frac{1}{\psi} = \gamma$), a risk aversion coefficient above one instead implies that the wealth effect dominates which results in a positive value of $A_{c,2}(s_t)$ when $\beta_2(s_t) < 0$. This has counterfactual implications since it implies rising asset prices in times of stagflation which is opposite to what we observe in data.

$A_{c,3}(s_t)$ is negative given a high value of the EIS indicating a negative relation between price-consumption ratios and inflation volatility in both regimes. As investors dislike uncertainty about growth in Bansal and Yaron (2004), investors dislike uncertainty about inflation in our specification.

In order to understand how macro shocks affect the well-being of investors we next consider innovations to the real pricing kernel. To do so, we take expectations using the information set $I_t = \{x_{c,t}, x_{\pi,t}, \sigma_{\pi,t}^2, s_t\}$ which means that the state tomorrow s_{t+1} is uncertain:

$$\begin{aligned}
m_{t+1}(s_{t+1}) - E[m_{t+1}(s_{t+1})|I_t] &= -\lambda_{\eta_c}\sigma_c\eta_{c,t+1} - \lambda_{\varepsilon_c}(s_{t+1})\sigma_c\varepsilon_{c,t+1} - \lambda_{\nu}(s_{t+1})\sigma_{\nu}w_{t+1} - \lambda_{\varepsilon_{\pi}}(s_{t+1})\sigma_{\pi,t}\varepsilon_{\pi,t+1} \\
&\quad + V(s_t, s_{t+1}) \tag{12} \\
\lambda_{\eta_c} &= \gamma \\
\lambda_{\varepsilon_c}(s_{t+1}) &= (1 - \theta)[k_{c,1}A_{c,1}(s_{t+1})\delta_1(s_{t+1}) + k_{c,1}A_{c,2}(s_{t+1})\delta_3(s_{t+1})] \\
\lambda_{\nu}(s_{t+1}) &= (1 - \theta)k_{c,1}A_{c,3}(s_{t+1}) \\
\lambda_{\varepsilon_{\pi}}(s_{t+1}) &= (1 - \theta)[k_{c,1}A_{c,1}(s_{t+1})\delta_2(s_{t+1}) + k_{c,1}A_{c,2}(s_{t+1})\delta_4(s_{t+1})],
\end{aligned}$$

where the λ 's represent market prices of risk and where the expression for $V(s_t, s_{t+1})$ arises because tomorrow's state is uncertain as of time t . We report the actual expression for $V(s_t, s_{t+1})$ in the appendix.

The first two sources of risk are the same as in Bansal and Yaron (2004), namely short run consumption risk and long-run consumption risk. The third shock term reflects shocks to inflation volatility. Volatility shocks are priced and has a negative price of risk ($\lambda_{\nu}(s_{t+1}) < 0$) regardless of economic regime. Hence, the representative agent dislikes higher inflation uncertainty in all states of the world. In contrast to existing long-run risk specifications, this setup also allows for

long-run inflation shocks to be priced which makes up the last shock term in (12). A negative value of $\lambda_{\varepsilon\pi}(s_t)$ implies that the representative agent dislikes positive shocks to expected inflation and therefore requires a positive risk premium on assets that perform badly in periods of high inflation. This was for example the case during the stagflationary period of 1970-1980. The crucial feature of this model is that $\lambda_{\varepsilon\pi}(s_t)$ can switch sign and take on a positive value. The sign depends on whether inflation and growth are positively or negatively correlated. $\lambda_{\varepsilon\pi}(s_t)$ is negative when inflation and growth are negatively correlated and positive when inflation is procyclical. Recall that $\theta = 1$ under power utility, which means that long-run inflation risk is not priced and that the only sources of priced risk left are short-run consumption risk λ_{η_c} . Overall, long-run inflation shocks represent an additional risk premium part in the economy compared to models in which only consumption shocks are priced.

The log price-dividend ratio is conjectured to be a linear function of the three state variables:

$$pd_t(s_t) = A_{d,0}(s_t) + A_{d,1}(s_t)x_{c,t} + A_{d,2}(s_t)x_{\pi,t} + A_{d,3}(s_t)\sigma_{\pi,t}^2. \quad (13)$$

The coefficients are solved in an analogous manner to the A_c coefficients. Appendix A.2 describes the derivations in detail and reports the full analytical expressions. Again, the expression for $A_{d,1}(s_t)$ collapses to the expression in Bansal and Yaron (2004) should we restrict the model to a single-regime economy. $A_{d,2}(s_t)$ and $A_{d,3}(s_t)$ are new compared to existing long-run risk models and determine the impact of the level of inflation and inflation volatility on equity prices.

Expected consumption growth and price-dividend ratios are positively associated for high values of the EIS meaning that $A_{d,1}(s_t) > 0$ in both regimes. $A_{d,2}(s_t)$ is in general negative when the EIS is above one and when inflation is bad for future growth $\beta_2(s_t) < 0$. This means that high expected inflation depresses equity valuation ratios in periods of countercyclical inflation. In periods of procyclical inflation, $\beta_2(s_t)$ is positive which switches the sign of $A_{d,2}(s_t)$ to positive. This mechanism allows the model to match the switching relation between price-dividend ratios and inflation levels found in data, as was shown in Figure 8 and Table 3.

A rise in inflation volatility has a negative impact on price-dividend ratios ($A_{d,3}(s_t) < 0$) in both regimes provided a high value of the EIS. While the relation between price-dividend ratios

and the level of inflation can switch sign in the model, a rise in inflation uncertainty is always bad news for equity valuations. This arises since inflation shocks always contribute to procyclical stock returns in the model regardless of whether inflation is counter or procyclical. Hence, equity is always risky with respect to inflation shocks. This is consistent with Figure 8 and Table 3 showing a negative relation between price-dividend ratios and inflation risk in data throughout the entire sample period.

4.5 Solving for Real Bonds

The log price of a real bond with a maturity of n periods is conjectured to be function of the same state variables as before:

$$q_{t,n}(s_t) = D_{0,n}(s_t) + D_{1,n}(s_t)x_{c,t} + D_{2,n}(s_t)x_{\pi,t} + D_{3,n}(s_t)\sigma_{\pi,t}^2. \quad (14)$$

Let $y_{t,n} = -\frac{1}{n}q_{t,n}$ denote the n -period continuously compounded real yield. Then:

$$y_{t,n}(s_t) = -\frac{1}{n}(D_{0,n}(s_t) + D_{1,n}(s_t)x_{c,t} + D_{2,n}(s_t)x_{\pi,t} + D_{3,n}(s_t)\sigma_{\pi,t}^2), \quad (15)$$

where the D-coefficients determine how yields respond to changes in expected consumption growth, expected inflation, and inflation volatility. Appendix A.3 shows how we solve for the coefficients and reports the expressions.

For plausible parameter values, real yields increase in response to higher expected consumption growth ($D_{1,n}(s_t) < 0$). Consumption shocks therefore generate countercyclical bond returns and contribute to negative risk premiums on real bonds. Real yields decrease in response to higher inflation when inflation is bad news for growth ($\beta_2(s_t) < 0$), resulting in a positive $D_{2,n}(s_t)$ coefficient. In this case, inflation shocks also contribute to negative expected returns since they generate positive bond returns in bad inflationary times. This is consistent with earlier studies such as Fama and Gibbons (1982), Pennacchi (1991), and Boudoukh (1993). Ang et al. (2008) also document a negative relation between real rates and expected inflation but find the correlation to be positive for longer horizons. Note that a switch in the sign of $\beta_2(s_t)$ to positive implies the opposite, namely

that real yields move positively with inflation ($D_{2,n}(s_t) < 0$). Hence, the model can accommodate changes in the relation between real interest rates and inflation via the $\beta_2(s_t)$ parameter.

An increase in inflation uncertainty lowers real yields ($D_{3,n}(s_t) > 0$) with long rates dropping more than short rates. This occurs because inflation risk moves real bonds through a discount-rate channel. When inflation is considered bad news for growth, inflation shocks lower real yields as discussed above and therefore generates high bond returns in bad times. If inflation instead is positively related to growth, inflation shocks raise real yields, generating poor bond returns in good times. In both cases, inflation shocks contribute to countercyclical returns and therefore to a negative risk premium on real bonds. Hence, a rise in inflation volatility is always associated with lower real yields regardless of the economic state.

4.6 Solving for Nominal Bonds

Nominal log bond prices are conjectured to be functions of the same state variables:

$$q_{t,n}^{\$}(s_t) = D_{0,n}^{\$}(s_t) + D_{1,n}^{\$}(s_t)x_{c,t} + D_{2,n}^{\$}(s_t)x_{\pi,t} + D_{3,n}^{\$}(s_t)\sigma_{\pi,t}^2. \quad (16)$$

Let $y_{t,n}^{\$} = -\frac{1}{n}q_{t,n}^{\$}$ denote the n -period continuously compounded nominal yield. Then:

$$y_{t,n}^{\$}(s_t) = -\frac{1}{n}(D_{0,n}^{\$}(s_t) + D_{1,n}^{\$}(s_t)x_{c,t} + D_{2,n}^{\$}(s_t)x_{\pi,t} + D_{3,n}^{\$}(s_t)\sigma_{\pi,t}^2), \quad (17)$$

where the $D^{\$}$ -coefficients determine how nominal yields respond to changes in expected consumption growth, inflation, and inflation volatility. Solving for nominal log bond prices requires the use of the nominal log pricing kernel which is determined by the difference between the real log pricing kernel and the inflation rate:

$$m_{t+1}^{\$}(s_{t+1}) = m_{t+1}(s_{t+1}) - \pi_{t+1}.$$

Appendix A.4 shows how to solve for the coefficients and reports the detailed expressions.

The response of nominal yields to changes in expected growth is the same as for real yields

meaning that $D_{1,n}^{\$}(s_t) < 0$ in both states for reasonable parameter values. This means that shocks to consumption growth contribute to negative risk premiums also for nominal bonds. As expected, nominal yields move positively with expected inflation implying a negative value of $D_{2,n}^{\$}(s_t)$. This holds regardless of economic state and reflects a cash-flow effect on nominal bonds. Hence, while real yields may decrease or increase in response to expected inflation depending on the current cyclical state, nominal yields always rise with the level of inflation, which is consistent with data.

The effect of inflation volatility on yields depends on whether inflation is counter or procyclical and reflects a discount-rate channel stemming from inflation risk. When inflation is negatively (positively) correlated with growth, higher inflation will raise yields through the cash-flow channel and generate poor bond returns in bad (good) times. Hence, nominal bonds may be subject to either countercyclical or procyclical returns and can therefore constitute both a risky asset or a hedging asset with respect to inflation risk. It all depends on the inflation regime. As a result, a rise in inflation risk can therefore be associated with both higher or lower yields through its different effect on bond risk premia. This means that $D_{3,n}^{\$}(s_t)$ can be either negative or positive depending on the inflation regime. This is the key mechanism that allows the model to match the switching relation between nominal interest rates and inflation uncertainty documented earlier. We elaborate further on the link between inflation risk and bond risk premia in Section 6.

5 Calibration of Model

5.1 Calibration

Motivated by the estimated breaking point in our empirical regime-switching model, namely 2001:2, we calibrate the model for the periods pre and post the second quarter of 2001. We target a range of unconditional macro and asset-pricing moments based on consumption growth, inflation, dividend growth, stock returns, and bond returns. We assume that the quarterly frequency of the model coincides with the decision interval of the agent. This means we abstract away from issues related to time-aggregation of consumption growth. The effect of time-aggregation does not affect the

qualitative results of the paper.¹⁸

We first describe how we calibrate the model and then we discuss the implied macro and asset-pricing moments. All calibrated parameters are reported in Table 10. Recall from the model specification that we only allow consumption and inflation parameters to depend on the regime. This means that dividend growth parameters, volatility parameters, and the preference parameters are kept constant across regimes. This is done for parsimonious reasons since our objective is to highlight the impact of the growth-inflation relation on asset prices. By restricting a number of parameters to be constant we make it harder for the model to match the empirical moments.

The mean parameters for growth and inflation, μ_c and μ_π , are set equal to their sample values for each sub-period. The mean of dividend growth, μ_d , is set equal to the full sample mean. The persistence of shocks to consumption growth, β_1 , is set to 0.951 and 0.995 for the two periods. These values translate into 0.983 and 0.998 on a monthly frequency. The first value is in line with existing calibrated and estimated values in the long-run risk literature while the second value is somewhat higher than established values. The reason for this difference is that we find the persistence of consumption growth to be substantially higher in the second period compared to the first.

The β_2 parameter is set to -0.012 and 0.012 respectively, which means that inflation expectations have a negative impact on future growth expectations in the countercyclical state but a positive impact in the procyclical period. This creates a negative (positive) correlation between growth and inflation in the countercyclical (procyclical) period and that we later show is in line with data. The different signs of β_2 for the two states also allows the model to match the switching relation between price-dividend ratios and expected inflation, from negative to positive, that we documented earlier. More specifically, the sign of β_2 determines the sign of $A_{d,2}$ in Equation (13). The last of the β parameters is β_4 which governs the persistence of inflation. We find in data that the persistence of inflation was substantially higher during the countercyclical period compared to the more recent procyclical period. We therefore calibrate β_4 to 0.90 and 0.40 respectively.

The next set of parameters refer to the δ matrix which governs the size of long-run shocks to growth and inflation. The parameters governing shocks to expected growth, δ_1 is set to 0.12 for

¹⁸See for example Bansal et al., 2007a, Bansal et al., 2007b, and Hasseltoft (2011) who all estimate long-run risk models using simulation estimators, taking into account time-aggregation of consumption growth.

both periods. The parameters δ_2 and δ_3 affect the correlation between growth and inflation and are set equal to -0.15 and 0.20 for δ_2 and to -0.10 and 0.9 for δ_3 . This helps the model to match the switch in macro correlations from a negative to a positive state. Lastly, δ_4 governs the size of long-run shocks to inflation and is calibrated to 0.90 for both state.

Overall, the main difference in parameter values across periods stems from matching the significant shift in the growth-inflation correlation from negative to positive plus matching the large increase in growth persistence and the large drop in inflation persistence that occurred during the 2000's.

The persistence of volatility shocks v_π and their volatility σ_ν are calibrated to standard values in the long-run risk literature, 0.98 and $1 * 10^{-6}$ respectively. Dividend parameters and the preference parameters are also calibrated to standard values in the literature. The risk aversion is set to 10 and the EIS to 2 . It is well-known that long-run risk models need an EIS above one in order to generate plausible asset pricing implications. The value of the EIS is subject to controversy. While for example Hall (1988), Campbell (1999), and Beeler and Campbell (2011) estimate the EIS to be close to zero, Attanasio and Weber (1993), Attanasio and Vissing-Jorgensen (2003), Chen et al. (2008), and Hasseltoft (2011) among others find the EIS to be above one. Lastly we need to calibrate the transition probabilities. We set them equal to the estimated values from the empirical regime-switching model, namely $p_{00} = 0.98$ and $p_{11} = 0.94$.

Having calibrated the model, we simulate the model 150000 quarters and evaluate the implied macro and asset-price moments. Table 11 reports moments for macro data. We report moments for the countercyclical period, the procyclical period, and for the full sample. The unconditional means are matched perfectly by construction. Volatility of the macroeconomic variables all lie close to their sample values. The table shows clearly that the persistence of growth increased sharply during 2000's with a first-order autocorrelation coefficient of 0.77 versus 0.39 for the countercyclical period. The persistence of inflation also changed markedly but in the opposite direction with a large drop in the first-order autocorrelation coefficient from 0.84 in the countercyclical state to 0.27 . Our calibration matches these sharp changes in persistence where inflation is substantially more persistent than growth pre 2001:2 but then substantially less persistent throughout the 2000's. We

report the fourth-order autocorrelation coefficient for dividend growth since the moving-average procedure described above automatically induces positive autocorrelation for up to three lags.

Table 12 contains unconditional asset-price moments. The calibration generates model moments that are broadly in line with data. While the level of the equity risk premium and price-dividend ratios are more or less in line with data, the volatilities are lower than in data. The level of the nominal short rate is matched well and the model generates an upward sloping nominal yield curve. However the model-implied slope of roughly 50 basis points is lower than observed in data. The model-implied real yield curve is downward sloping in the countercyclical period and flat in the procyclical period.

Table 13 reports various macro and asset correlations. First, the correlation between real growth and inflation changed from negative to positive in data, -0.42 versus 0.30 . The model is able to match this shift. Second, the correlation between dividend yields and nominal yields changed sharply in data from 0.68 to -0.65 . The model is able to generate a similar shift from positive to negative correlations, albeit the correlation in the procyclical is more negative than in data. Third, the correlation between stock and bond returns changed substantially from positive to negative in data, 0.28 versus -0.63 . The model matches this transition well.

Overall, we believe the model does a good job in matching key moments. The model manages in particular to match the large shift in the growth-inflation correlation from negative to positive and the contemporaneous shift in asset correlations from positive to negative. The model could of course do an even better job if we relaxed the many imposed restrictions. Our objective, however, has been to match the broad change in macro and asset-prices across the two regimes while keeping the model as parsimonious as possible.

6 Asset Pricing Implications

Given our calibrated parameters, we here discuss in detail the model-implications for bonds, equity, and asset correlations.

6.1 Nominal Bonds

Consider the innovation to nominal yields:

$$\begin{aligned}
y_{t+1,n}^{\$}(s_{t+1}) - E[y_{t+1,n}^{\$}(s_{t+1})|I_t] &= -\frac{1}{n} \left[\sigma_c \varepsilon_{c,t+1} [D_{1,n-1}^{\$}(s_{t+1})\delta_1(s_{t+1}) + D_{2,n-1}^{\$}(s_{t+1})\delta_3(s_{t+1})] \right. \\
&+ \sigma_{\pi,t} \varepsilon_{\pi,t+1} [D_{1,n-1}^{\$}(s_{t+1})\delta_2(s_{t+1}) + D_{2,n-1}^{\$}(s_{t+1})\delta_4(s_{t+1})] \\
&\left. + \sigma_v w_{t+1} D_{3,n-1}^{\$}(s_{t+1}) \right] + Q(s_t, s_{t+1}), \tag{18}
\end{aligned}$$

where $Q(s_t, s_{t+1})$ is reported in the appendix and arises from the fact that tomorrow's regime s_{t+1} is uncertain. Let us focus on the economically interesting shock terms. The response of nominal rates to consumption shocks $\varepsilon_{c,t+1}$, depends on $D_{1,n}^{\$}(s_{t+1})\delta_1(s_{t+1})$ which is negative in both inflation regimes and $D_{2,n}^{\$}(s_{t+1})\delta_3(s_{t+1})$ which is positive in the countercyclical state and negative in the procyclical state. Overall, the term in front of $\sigma_c \varepsilon_{c,t+1}$ is negative in both inflation regimes meaning that nominal yields increase in response to higher consumption growth. The second term in (18) determines how nominal rates move with shocks to expected inflation. The term $D_{1,n}^{\$}(s_{t+1})\delta_2(s_{t+1})$ is positive in the countercyclical state and negative in the procyclical regime reflecting the different effects inflation shocks have on growth in the two regimes. $D_{2,n}^{\$}(s_{t+1})\delta_4(s_{t+1})$ on the other hand is highly negative in both states. Overall, the expression in front of inflation shocks $\sigma_{\pi,t} \varepsilon_{\pi,t+1}$ is highly negative in both states implying that nominal yields move positively with expected inflation. This is consistent with economic intuition and represents a cash-flow effect on nominal bonds.

The effect of shocks to inflation volatility on yields is positive ($D_{3,n-1}^{\$}(s_{t+1}) < 0$) and increasing with maturity in the countercyclical state. However the effect is reversed for the procyclical state ($D_{3,n-1}^{\$}(s_{t+1}) > 0$) leading to lower yields as inflation uncertainty increases. Movements in $\sigma_{\pi,t}^2$ affects yields through a discount-rate channel. Expected excess returns on nominal bonds can move both positively or negatively with inflation uncertainty which means that shocks to inflation uncertainty can move yields both up or down. To see this more clearly, consider next risk premiums on nominal bonds.

First, positive shocks to expected growth means good times for the agent but low bond returns. As a result, shocks to consumption growth contribute to a negative risk premium. This holds for

both the countercyclical and procyclical state. Second, higher inflation signals bad times ahead for the agent in the countercyclical state at the same time as bond returns are low. This results in procyclical bond returns and nominal bonds are therefore risky assets with respect to inflation shocks. In contrast, higher inflation in the procyclical state signals good times for the agent. Returns on nominal bonds are still affected negatively by an increase in inflation but returns are then countercyclical. Hence, nominal bonds provide a hedge against inflation risk in the procyclical regime.

To be more specific, let $h_{t+1,n}^{\$}(s_{t+1}) = q_{t+1,n-1}^{\$}(s_{t+1}) - q_{t,n}^{\$}(s_t)$ denote the one period log holding period return on a nominal bond with a maturity of n periods. The risk premium can then be written as:

$$\begin{aligned}
E[h_{t+1,n}^{\$}(s_{t+1}) - r_{f,t}^{\$} | I_t] &+ \frac{1}{2} \sum_{s_{t+1}=0,1} p_{s_t,s_{t+1}} \text{Var}[h_{t+1,n}^{\$}(s_{t+1}) | I_{t+1}] \\
&= \sum_{s_{t+1}=0,1} p_{s_t,s_{t+1}} [A(s_{t+1}) + B(s_{t+1})\sigma_{\pi,t}^2], \\
B(s_{t+1}) &= [D_{1,n-1}^{\$}(s_{t+1})\delta_2(s_{t+1}) + D_{2,n-1}^{\$}(s_{t+1})\delta_4(s_{t+1})]\lambda_{\varepsilon_{\pi}}(s_{t+1}),
\end{aligned} \tag{19}$$

and where $A(s_{t+1})$ is reported in the appendix for brevity. Time-varying volatility of inflation gives rise to a time-varying risk premium on nominal bonds. The $B(s_{t+1})$ coefficient is determined by the market price of long-run inflation risk $\lambda_{\varepsilon_{\pi}}(s_{t+1})$ times the response of bond prices to inflation shocks. Recall from Equation (18) that nominal yields move positively with inflation across both states, meaning that $(D_{1,n-1}^{\$}(s_{t+1})\delta_2(s_{t+1}) + D_{2,n-1}^{\$}(s_{t+1})\delta_4(s_{t+1})) < 0$ in both states. However, the market price of inflation risk changes sign between the different states. In the countercyclical period, investors dislike inflation shocks so the market price of inflation is negative. Therefore risk premiums load positively on inflation volatility, $B(s_{t+1}) > 0$, in the procyclical regime. However, the market price inflation shocks turns positive in the procyclical state which means risk premiums move negatively with inflation uncertainty. Conventional wisdom tend to suggest that high uncertainty about inflation should result in higher risk premiums on nominal bonds. This only holds if inflation is bad for economic growth. If inflation instead is associated with high growth, high inflation uncertainty in fact lowers bond risk premiums. The model suggests this was the case for the period

2001:2-2011:4.

According to the model, investors dislike inflation shocks during stagflationary periods at the same time as nominal bond returns are procyclical. As a result, nominal bonds are risky assets with respect to inflation risk and bond risk premiums and yields should therefore move positively with inflation uncertainty. This is in line with data as was reported earlier in Table 4. However, the same table showed that the opposite was true during the recent 10 years. The model can explain why inflation uncertainty and nominal yields suddenly started to comove negatively. A positive comovement of inflation and growth implies a positive market price of inflation risk in the model and countercyclical bond returns. Nominal bonds therefore serve as insurance against inflation risk which means a jump in inflation uncertainty should lower expected returns on bonds and their yields. Note that the effect of movements in inflation uncertainty on yields is distinct from changes in the level of inflation. An increase in the level of expected inflation represents a cash-flow effect on bonds and is always bad for bond returns and raises yields. This is implied by the model and is also what we observe in data.

Table 14 shows results from regressing the level of 5-year nominal rates onto expected inflation and inflation uncertainty inside the model. We simulate 150000 quarters and run regressions for the full simulated sample, for the countercyclical period of the sample, and for the procyclical period of the sample. First of all, model yields load positively on the level of expected inflation for all three samples which is expected. The slope coefficients for inflation uncertainty are more interesting. They suggest that model-implied rates load positively on inflation uncertainty for the full sample and for the countercyclical period. However, the coefficient on inflation uncertainty switches sign in the procyclical state. An increase in inflation uncertainty in fact *lowers* bond yields through its negative impact on expected returns. A change in the sign of the market price of inflation risk makes the model able to replicate the switching relation between inflation risk and nominal yields that is present in data. Overall, we believe these results indicate that it is important for any asset-pricing model that prices nominal bonds to account for the changing cyclical nature of inflation.

6.2 Equity

Consider the innovation to log price-dividend ratios:

$$\begin{aligned}
 pd_{t+1}(s_{t+1}) - E[pd_{t+1}(s_{t+1})|I_t] &= \sigma_c \varepsilon_{c,t+1} [A_{d,1}(s_{t+1})\delta_1(s_{t+1}) + A_{d,2}(s_{t+1})\delta_3(s_{t+1})] \\
 &+ \sigma_{\pi,t} \varepsilon_{\pi,t+1} [A_{d,1}(s_{t+1})\delta_2(s_{t+1}) + A_{d,2}(s_{t+1})\delta_4(s_{t+1})] \\
 &+ \sigma_v w_{t+1} A_{d,3}(s_{t+1}) + S(s_t, s_{t+1}), \tag{20}
 \end{aligned}$$

where $S(s_t, s_{t+1})$ is reported in the appendix and arises from the fact that tomorrow's state s_{t+1} is uncertain. Given an EIS above one, the two terms $A_{d,1}(s_{t+1})\delta_1(s_{t+1})$ and $A_{d,2}(s_{t+1})\delta_3(s_{t+1})$ are positive in both inflation regimes. This means that price-dividend ratios move positively with economic growth regardless of the current regime.

Our focus is more on the second term which represents the impact of inflation shocks on price-dividend ratios. Recall that a negative relation between inflation expectations at time t and future growth expectations ($\beta_2(s_{t+1}) < 0$) implies a negative relation between expected inflation and price-dividend ratios, ($A_{d,2}(s_{t+1}) < 0$). This is the case for the countercyclical state in which inflation shocks depress equity-valuation ratios. This effect is further amplified by the term $A_{d,1}(s_{t+1})\delta_2(s_{t+1})$ which also is negative in the countercyclical state. As a result, the entire term in front of inflation shocks $\sigma_{\pi,t} \varepsilon_{\pi,t+1}$ is negative. Intuitively, price-dividend ratios decrease in response to inflation shocks when those shocks are bad news for growth. However the impact of inflation switches to positive in the procyclical state. A positive relation between inflation and growth therefore induces a positive relation between equity valuation ratios and inflation.

As discussed in Section 4.4, the effect of inflation volatility on price-dividend ratios, $A_{d,3}$ is always negative. If higher inflation is considered bad news by the agent and lead to low equity returns, equity returns are procyclical and therefore risky assets. An increase in inflation risk should therefore raise equity risk premiums and lower price-dividend ratios. If inflation instead represents good news for growth, stock returns will also be high which again produces procyclical stock returns. High returns in good times again implies that equity is risky with respect to inflation shocks. This suggests that risk premiums on equity should consistently load positive on inflation

uncertainty.

Table 14 shows results from regressing the log price-dividend ratio onto expected inflation and inflation uncertainty on simulated data inside the model. Results show that price-dividend ratios load negatively on both expected inflation and inflation uncertainty for the full sample period. This is consistent with the voluminous literature that documents equity being a poor inflation hedge. Coefficients are similar for the countercyclical sample period. However, the coefficient on expected inflation switches sign in the procyclical state. The model suggests that equity prices respond differently to changes in the level of inflation as opposed to changes in inflation uncertainty. A procyclical inflation induces a positive relation between inflation levels and stock prices. This can be interpreted as a cash-flow effect on stocks since higher growth in the model feeds into higher dividend growth. On the other hand, an increase in inflation uncertainty is always bad for stock prices since stock returns tend to always be procyclical. The model qualitatively matches the empirical regression results that were reported in Table 3.

Let us now analyze the equity risk premium. Let $r_{m,t+1}(s_{t+1})$ denote the one period log market return. The equity risk premium can then be written as:

$$\begin{aligned}
E[r_{m,t+1}(s_{t+1}) - r_{f,t}|I_t] &+ \frac{1}{2} \sum_{s_{t+1}=0,1} p_{s_t,s_{t+1}} \text{Var}[r_{m,t+1}(s_{t+1})|I_{t+1}] & (21) \\
&= \sum_{s_{t+1}=0,1} p_{s_t,s_{t+1}} [A(s_{t+1}) + B(s_{t+1})\sigma_{\pi,t}^2], \\
B(s_{t+1}) &= [k_{d,1}A_{d,1}(s_{t+1})\delta_2(s_{t+1}) + k_{d,1}A_{d,2}(s_{t+1})\delta_4(s_{t+1})]\lambda_{\varepsilon_{\pi}}(s_{t+1}),
\end{aligned}$$

and where the $A(s_{t+1})$ coefficient is reported in the appendix for brevity. As with bonds, risk premiums on equity vary with inflation volatility. The $B(s_{t+1})$ coefficient is determined by the market price of long-run inflation risk $\lambda_{\varepsilon_{\pi}}(s_{t+1})$ times the impact of inflation shocks on real equity returns. In the countercyclical state, the agent dislikes inflation shocks so the market price of inflation is negative. At the same time inflation is bad for stock returns which means $[k_{d,1}A_{d,1}(s_{t+1})\delta_2(s_{t+1}) + k_{d,1}A_{d,2}(s_{t+1})\delta_4(s_{t+1})]$ is also negative. Low returns in bad times implies that the equity risk premium moves positively with inflation uncertainty.

Next consider the procyclical state. In this case, inflation is associated with good times so the

market price of inflation is positive. Now inflation is positively related to stock returns so the term $[k_{d,1}A_{d,1}(s_{t+1})\delta_2(s_{t+1}) + k_{d,1}A_{d,2}(s_{t+1})\delta_4(s_{t+1})]$ is also positive. High stock returns in good times means that stocks are risky assets also in this state. Consistent with our empirical evidence, risk premiums on equity is consistently positively related to inflation uncertainty.

6.3 Asset Correlations

This section describes model implications for the correlation between stock and bond returns and for the correlation between dividend yields and nominal yields. Both of these correlations changed sign dramatically in the early 2000's as was shown in Figures 2 and 5. We show below that the model can account for this switch by accounting for the changing cyclicity of inflation.

6.3.1 Stock and Bond Returns

We show in the appendix that the conditional covariance between stock and bond returns can be written as:

$$\begin{aligned}
 Cov[r_{m,t+1}^{\$}(s_{t+1}), h_{t+1,n}^{\$(s_{t+1})}|I_t] &= M_t + A + B\sigma_{\pi,t}^2, & (22) \\
 B &= p_{st,0}[k_{d,1}A_{d,1}^0\delta_2^0 + k_{d,1}A_{d,2}^0\delta_4^0][D_{1,n-1}^{\$,0}\delta_2^0 + D_{2,n-1}^{\$,0}\delta_4^0] \\
 &+ p_{st,1}[k_{d,1}A_{d,1}^1\delta_2^1 + k_{d,1}A_{d,2}^1\delta_4^1][D_{1,n-1}^{\$,1}\delta_2^1 + D_{2,n-1}^{\$,1}\delta_4^1],
 \end{aligned}$$

and where the M_t and A coefficients are reported in the appendix. We choose to focus on the B coefficient since it is largest in magnitude and determines how the stock-bond covariance moves with inflation uncertainty. The B -coefficient is basically a probability weighted measure of the impact of inflation shocks on price-dividend ratios and nominal bonds in the two regimes. Based on the earlier discussion, we know that inflation shocks impact price-dividend ratios negatively in the countercyclical state and positively in the procyclical state. For nominal bonds we know that positive inflation shocks lower bond prices regardless of economic state. It is then evident from Equation (22) that the conditional covariance moves positively with inflation uncertainty when inflation is bad for economic growth and negatively with inflation uncertainty when inflation is

procyclical. One way to interpret this comovement is to think about how inflation uncertainty moves equity and bond risk premia. Higher inflation uncertainty always raises expected returns on equity but move expected bond returns either up or down depending on the inflation regime.

We saw earlier in Table 5 that inflation uncertainty, in data, predicted the quarterly stock-bond covariance positively during the countercyclical state but negatively during the procyclical period. We would like to simulate our model and run the same regressions inside the model. However we cannot generate realized quarterly covariances in the model since the model is formulated on a quarterly frequency. Instead we report the value of the analytical B coefficient above. Table 14 reports that B is positive in the countercyclical state but switches to negative in the procyclical state. The model can match the switching behavior in data due to changes in the cyclical nature of inflation

Next we would like to plot the model-implied conditional correlation between stock and bond returns. In order to do so we need empirical proxies for our state variables. We construct expected growth and inflation by projecting realized values onto a set of instruments and treat the fitted values as our state variables. Conditional variance of inflation is constructed by estimating an AR(1)-GARCH(1,1) on expected inflation.

Figure 12 plots the correlations. Consistent with data, the model implies highly positive correlations throughout the 1970's and early 1980's. We observe a drop in correlation around 1987 in the model since we earlier estimated inflation to briefly enter a procyclical period at that time. This drop is consistent with the drop in realized correlations seen in Figure 1. The drop in model correlations is more extreme since realized correlations are computed as 5-year rolling correlations which smooths out large changes. As we approach the end of the 1990's correlations start to turn lower and drops sharply in the early 2000 as we enter the procyclical region. Correlations then stay highly negative throughout the 2000's except for a sharp spike towards the end of the sample. Figure 1 shows a similar spike in realized correlations around the same period. Overall, the model seems to provide a good fit to the general trend in correlations.

6.3.2 Dividend Yields and Nominal Yields

Since equity returns are closely related to changes in dividend yields and bond returns to changes in yields, the same mechanism can be used to explain why dividend yields and nominal yields comove. The existing literature focuses on the highly positive correlation between these two variables between 1960 and 2000. As mentioned earlier, this observation is often dubbed the Fed-model. However, it is rarely mentioned in the literature that this correlation changed dramatically during the last 10 years, from a correlation of 0.64 during 1965:1-2001:2 to a correlation of -0.57 during 2001:3-2011:4. The behavioral concept of inflation illusion has been extensively used in the literature to explain the positive comovement. However, the inflation illusion story has problems of explaining the significant shift to negative correlations. We find that our model can provide a rational explanation for why the comovement of dividend yields and nominal yields changes over time.

Consider the expression for the conditional covariance between dividend yields and nominal yields:

$$Cov[pd_{t+1}(s_{t+1}), y_{t+1,n}^{\$}(s_{t+1})|I_t] = M_t + A + B\sigma_{\pi,t}^2, \quad (23)$$

$$B = -\frac{1}{n} \left[p_{s_t,0} [A_{d,1}^0 \delta_2^0 + A_{d,2}^0 \delta_4^0] [D_{1,n-1}^{\$,0} \delta_2^0 + D_{2,n-1}^{\$,0} \delta_4^0] \right. \\ \left. + p_{s_t,1} [A_{d,1}^1 \delta_2^1 + A_{d,2}^1 \delta_4^1] [D_{1,n-1}^{\$,1} \delta_2^1 + D_{2,n-1}^{\$,1} \delta_4^1] \right],$$

and where M_t and A are reported in the appendix. We focus on the B-coefficient since it is largest in magnitude and provide intuition of how the covariance moves with inflation uncertainty. Comparing the B-coefficient with the one in Equation (22), it is evident that they are very similar. The same argument as for the stock-bond covariance goes through for explaining the so called Fed-model. Changes in inflation uncertainty moves equity and bond risk premiums in the same direction when inflation is countercyclical. However inflation uncertainty moves stock and bond risk premia in opposite directions when inflation is procyclical since nominal bonds then provide

insurance against bad times while equity is still risky. As a result, we argue that movements in inflation risk together with changes in the cyclical nature of inflation can provide a plausible explanation for why dividend yields and nominal yields sometimes comove and sometimes diverge.

7 Conclusion

The correlation between returns on US stocks and Treasury bonds and the relation between inflation and asset prices have varied substantially over time. For example, the 1970-1980's was characterized by a highly positive correlation between stock and bond returns and a strong negative relation between inflation and price-dividend ratios. In contrast, the period 2000-2011 experienced the exact opposite with strongly negative asset correlations and a positive relation between inflation and equity valuations. We show that these observations line up remarkably well with the considerable time variation in the relation between consumption growth and inflation going back to the 1930's. While the 1970-1980's was characterized by stagflation, we show that inflation switched to a procyclical state in the early 2000's. We document in data that inflation risk is always negatively related to stock prices but can either decrease or increase bond prices depending on whether inflation is counter or procyclical. In countercyclical inflation regimes, nominal bonds are risky assets and therefore perform badly as inflation risk increases. However, nominal bonds provide a hedge against bad times when inflation is procyclical. This produces a drop in nominal rates as inflation risk increases, generating positive bond returns. We argue this asymmetry in how inflation risk impacts asset prices helps explain why the stock-bond correlation switches sign over time.

We calibrate a long-run risk model that illustrates the connection between the cyclicity of inflation and the joint movements of bond and equity risk premia and inflation and asset prices. Persistent inflation shocks have real effects and affects both equity and bond risk premia. We introduce a Markov-switching regime mechanism into the model which allows the relation between real growth and inflation to switch sign. Equity and bond risk premia are both functions of inflation volatility in the model but the loading of bond risk premia on inflation uncertainty depends on the cyclicity of inflation and can therefore switch sign. The model suggests that both equity and nominal bonds are risky assets when inflation is countercyclical, leading to a positive comovement

of asset risk premia in response to changes in inflation uncertainty. In contrast, nominal bonds provide a hedge against inflation risk when inflation is procyclical while equity is still risky. This implies that a rise in inflation uncertainty drives equity and bond risk premia in different directions, causing their returns to correlate negatively.

The model presented here is a first step in highlighting the importance of properly modeling changes in the cyclical nature of inflation. There are a number of directions in which these insights can be taken. First, the model is silent on the underlying determinants of inflation cyclical nature. What is the profound structural nature of shocks that characterize a procyclical or countercyclical state? What role does monetary policy play for understanding the joint dynamics of growth and inflation? Both our theoretical and empirical knowledge of this is limited.

8 Appendix

Model Specification

The processes for log consumption growth (g_{t+1}), log inflation (π_{t+1}), log dividend growth (Δd_{t+1}) and variance of inflation ($\sigma_{\pi,t+1}^2$) are given by:

$$\Delta c_{t+1} = \mu_c(s_{t+1}) + x_{c,t} + \sigma_c \eta_{c,t+1} \quad (24)$$

$$\pi_{t+1} = \mu_\pi(s_{t+1}) + x_{\pi,t} + \sigma_{\pi,t} \eta_{\pi,t+1} \quad (25)$$

$$\Delta d_{t+1} = \mu_d + \phi x_{c,t} + \varphi \sigma_c \eta_{d,t+1} \quad (26)$$

$$\sigma_{\pi,t+1}^2 = \sigma_\pi^2 + v_1(\sigma_{\pi,t}^2 - \sigma_\pi^2) + \sigma_v w_{t+1} \quad (27)$$

The dynamics for the time-varying parts of the conditional means of the above processes are given by:

$$x_{c,t+1} = \beta_1(s_{t+1})x_{c,t} + \beta_2(s_{t+1})x_{\pi,t} + \delta_1(s_{t+1})\sigma_c \varepsilon_{c,t+1} + \delta_2(s_{t+1})\sigma_{\pi,t} \varepsilon_{\pi,t+1} \quad (28)$$

$$x_{\pi,t+1} = \beta_4(s_{t+1})x_{\pi,t} + \delta_3(s_{t+1})\sigma_c \varepsilon_{c,t+1} + \delta_4(s_{t+1})\sigma_{\pi,t} \varepsilon_{\pi,t+1} \quad (29)$$

All shocks are mutually uncorrelated and i.i.d. normally distributed with a mean of zero and unit variance. $\beta_1(s_{t+1})$, $\beta_2(s_{t+1})$, $\beta_4(s_{t+1})$, $\delta_1(s_{t+1})$, $\delta_2(s_{t+1})$, $\delta_3(s_{t+1})$, and $\delta_4(s_{t+1})$, which govern the persistence of consumption and inflation shocks and their effect on the conditional mean of consumption growth and inflation, as well as the $\mu_c(s_{t+1})$ and $\mu_\pi(s_{t+1})$ depend on tomorrow's regime $s_{t+1} = (0, 1)$. The probability of ending up in tomorrow's regime $s_{t+1} = (0, 1)$ given today's regime $s_t = (0, 1)$ is governed by the transition probability matrix of a Markov chain:

$$P = \begin{bmatrix} p_{00} & p_{10} \\ p_{01} & p_{11} \end{bmatrix},$$

where $P(s_{t+1} = j | s_t = i) = p_{ij}$, $\sum_{j=0}^1 p_{ij} = 1$ and $0 < p_{ij} < 1$. We assume that agents can observe the current regime.

The log IMRS for Epstein-Zin preferences can be written as:

$$m_{t+1}(s_{t+1}) = \theta \ln(\delta) - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1}(s_{t+1}) \quad (30)$$

Solving the Model:

In the following Sections A.1 - A.4 we show how to solve the model using approximate analytical solutions.

A.1 The Price-Consumption Ratio

The return on the unobservable aggregate wealth portfolio is approximated as in Campbell and Shiller (1988):

$$r_{c,t+1}(s_{t+1}) = k_{c,0} + k_{c,1} p_{c,t+1}(s_{t+1}) - p_{c,t}(s_t) + \Delta c_{t+1}, \quad (31)$$

where $p_{c,t}$ denotes the log price-consumption ratio. The constants k_c are functions of the average level of $p_{c,t}$, which we denote by $\bar{p}c$.¹⁹

The log price-consumption ratio is conjectured to be a linear function of our state variables²⁰:

$$p_{c,t}(s_t) = A_{c,0}(s_t) + A_{c,1}(s_t)x_{c,t} + A_{c,2}(s_t)x_{\pi,t} + A_{c,3}(s_t)\sigma_{\pi,t}^2 \quad (32)$$

¹⁹ $k_{c,1} = \frac{\exp(\bar{p}c)}{1 + \exp(\bar{p}c)}$ and $k_{c,0} = \ln(1 + \exp(\bar{p}c)) - k_{c,1}\bar{p}c$.

²⁰ Which are the time-varying conditional means of consumption growth and inflation and the time-varying conditional variance of inflation.

The A coefficients governing the price-consumption ratio can be derived using the log IMRS together with the given macro dynamics and the approximation for the return on the aggregate wealth portfolio. We will make use of the law of iterated expectations and of the Euler equation for the consumption asset, which can be written as:

$$E[\exp\{m_{t+1}(s_{t+1}) + r_{c,t+1}(s_{t+1})\}|I_t] = 1. \quad (33)$$

We first form expectations using the information set $I_{t+1} = \{x_{c,t}, x_{\pi,t}, \sigma_{\pi,t}^2, s_t, s_{t+1}\}$ and then condition them down by using the current information set $I_t = \{x_{c,t}, x_{\pi,t}, \sigma_{\pi,t}^2, s_t\}$. Using the law of iterated expectations, the Euler equation for the consumption asset can then be rewritten as:

$$1 = E[\exp\{m_{t+1}(s_{t+1}) + r_{c,t+1}(s_{t+1})\}|I_t] \quad (34)$$

$$= E[E[\exp\{m_{t+1}(s_{t+1}) + r_{c,t+1}(s_{t+1})\}|I_{t+1}]|I_t] \quad (35)$$

$$= \sum_{s_{t+1}=0,1} p_{s_t, s_{t+1}} E[\exp\{m_{t+1}(s_{t+1}) + r_{c,t+1}(s_{t+1})\}|I_{t+1}] \quad (36)$$

Making use of the conditional normality (given I_{t+1}) of log consumption growth and the state variables (and therefore also $r_{c,t+1}$ and m_{t+1}) in a first step, and of the approximation $e^y - 1 \approx y$ in the second step²¹, the above Euler condition can be restated as:

$$1 = \sum_{s_{t+1}=0,1} p_{s_t, s_{t+1}} \exp\{E[m_{t+1}(s_{t+1}) + r_{c,t+1}(s_{t+1})|I_{t+1}] + \frac{1}{2}Var[m_{t+1}(s_{t+1}) + r_{c,t+1}(s_{t+1})|I_{t+1}]\} \quad (37)$$

$$0 = \sum_{s_{t+1}=0,1} p_{s_t, s_{t+1}} \{E[m_{t+1}(s_{t+1}) + r_{c,t+1}(s_{t+1})|I_{t+1}] + \frac{1}{2}Var[m_{t+1}(s_{t+1}) + r_{c,t+1}(s_{t+1})|I_{t+1}]\} \quad (38)$$

²¹This approximation has also been used in for example Bansal and Zhou (2002).

The conditional mean and the conditional variance in the above expression (38) are given by:

$$\begin{aligned}
E[m_{t+1}(s_{t+1}) + r_{c,t+1}(s_{t+1})|I_{t+1}] &= \theta \ln(\delta) + \mu_c^{s_{t+1}} \overbrace{\left(\theta - \frac{\theta}{\psi}\right)}{=1-\gamma} + \theta(k_{c,0} - A_{c,0}^{s_t} + k_{c,1}\{A_{c,0}^{s_{t+1}} + A_{c,3}^{s_{t+1}}\sigma_\pi^2[1 - v_1]\}) \\
&+ x_{c,t}\left[\left(\theta - \frac{\theta}{\psi}\right) + \theta(k_{c,1}A_{c,1}^{s_{t+1}}\beta_1^{s_{t+1}} - A_{c,1}^{s_t})\right] \\
&+ x_{\pi,t}\left[\theta(k_{c,1}A_{c,1}^{s_{t+1}}\beta_2^{s_{t+1}} + k_{c,1}A_{c,2}^{s_{t+1}}\beta_4^{s_{t+1}} - A_{c,2}^{s_t})\right] \\
&+ \sigma_{\pi,t}^2[\theta(k_{c,1}A_{c,3}^{s_{t+1}}v_1 - A_{c,3}^{s_t})] \tag{39}
\end{aligned}$$

$$\text{Var}[m_{t+1}(s_{t+1}) + r_{c,t+1}(s_{t+1})|I_{t+1}] = \sigma_c^2 X^{s_{t+1}} + \sigma_{\pi,t}^2 Y^{s_{t+1}} + \sigma_v^2 Z^{s_{t+1}} \tag{40}$$

$$X^{s_{t+1}} = \left(\theta - \frac{\theta}{\psi}\right)^2 + (\theta k_{c,1}[A_{c,1}^{s_{t+1}}\delta_1^{s_{t+1}} + A_{c,2}^{s_{t+1}}\delta_3^{s_{t+1}}])^2 \tag{41}$$

$$Y^{s_{t+1}} = (\theta k_{c,1}[A_{c,1}^{s_{t+1}}\delta_2^{s_{t+1}} + A_{c,2}^{s_{t+1}}\delta_4^{s_{t+1}}])^2 \tag{42}$$

$$Z^{s_{t+1}} = (\theta k_{c,1}A_{c,3}^{s_{t+1}})^2 \tag{43}$$

Exploiting the fact that Equation ((38)) must hold for both starting regimes $s_t = (0, 1)$ and for all values of our state variables gives us a system of 8 equations and 8 unknowns and allows us to solve for the A_c -coefficients:

$$A_{c,1}^0 = \left(1 - \frac{1}{\psi}\right) \left[\frac{1 + \beta_1^1 k_{c,1} (p_{01} - p_{11})}{(1 - \beta_1^0 k_{c,1})(1 - p_{11} \beta_1^1 k_{c,1}) + p_{01}(\beta_1^0 k_{c,1}[1 - \beta_1^1 k_{c,1}])} \right] \quad (44)$$

$$A_{c,1}^1 = \left(1 - \frac{1}{\psi}\right) \left[\frac{1 + \beta_1^0 k_{c,1} (p_{01} - p_{11})}{(1 - \beta_1^0 k_{c,1})(1 - p_{11} \beta_1^1 k_{c,1}) + p_{01}(\beta_1^0 k_{c,1}[1 - \beta_1^1 k_{c,1}])} \right] \quad (45)$$

$$A_{c,2}^0 = \frac{\beta_2^1 A_{c,1}^1 k_{c,1} p_{01} + \beta_2^0 A_{c,1}^0 k_{c,1} [p_{00} + \beta_4^1 k_{c,1} (p_{01} - p_{11})]}{(1 - \beta_4^0 k_{c,1}) + p_{01}[\beta_4^0 k_{c,1} (1 - \beta_4^1 k_{c,1})] + p_{11}[\beta_4^1 k_{c,1} (\beta_4^0 k_{c,1} - 1)]} \quad (46)$$

$$A_{c,2}^1 = \frac{\beta_2^0 A_{c,1}^0 k_{c,1} p_{10} + \beta_2^1 A_{c,1}^1 k_{c,1} [p_{11} + \beta_4^0 k_{c,1} (p_{01} - p_{11})]}{(1 - \beta_4^0 k_{c,1}) + p_{01}[\beta_4^0 k_{c,1} (1 - \beta_4^1 k_{c,1})] + p_{11}[\beta_4^1 k_{c,1} (\beta_4^0 k_{c,1} - 1)]} \quad (47)$$

$$A_{c,3}^0 = \frac{1}{2} \frac{1}{\theta(1 - k_{c,1} v_1)} [p Y^{s_{t+1}=0} + (1 - p) Y^{s_{t+1}=1}] \quad (48)$$

$$A_{c,3}^1 = \frac{1}{2} \frac{1}{\theta(1 - k_{c,1} v_1)} [P Y^{s_{t+1}=0} + (1 - P) Y^{s_{t+1}=1}], \quad (49)$$

with the probabilities p and P given by:

$$p = \frac{1 - p_{01} + k_{c,1} v_1 [p_{01} - p_{11}]}{1 + k_{c,1} v_1 [p_{01} - p_{11}]} \quad (50)$$

$$1 - p = \frac{p_{01}}{1 + k_{c,1} v_1 [p_{01} - p_{11}]} \quad (51)$$

$$P = \frac{1 - p_{11}}{1 + k_{c,1} v_1 [p_{01} - p_{11}]} \quad (52)$$

$$1 - P = \frac{p_{11} + k_{c,1} v_1 [p_{01} - p_{11}]}{1 + k_{c,1} v_1 [p_{01} - p_{11}]} \quad (53)$$

$A_{c,0}^0$ and $A_{c,0}^1$ finally are given by:

$$A_{c,0}^0 = \frac{1}{\theta(1 - k_{c,1})} [\theta \ln(\delta) + \theta k_{c,0} + r W^{s_{t+1}=1} + (1 - r) W^{s_{t+1}=0}] \quad (54)$$

$$A_{c,0}^1 = \frac{1}{\theta(1 - k_{c,1})} [\theta \ln(\delta) + \theta k_{c,0} + R W^{s_{t+1}=1} + (1 - R) W^{s_{t+1}=0}], \quad (55)$$

where

$$W^{s_{t+1}=1} = \mu_c^1\left(\theta - \frac{\theta}{\psi}\right) + \theta k_{c,1} A_{c,3}^1 \sigma_\pi^2 [1 - v_1] + \frac{1}{2} \sigma_v^2 Z^{s_{t+1}=1} + \frac{1}{2} \sigma_c^2 X^{s_{t+1}=1} \quad (56)$$

$$W^{s_{t+1}=0} = \mu_c^0\left(\theta - \frac{\theta}{\psi}\right) + \theta k_{c,1} A_{c,3}^0 \sigma_\pi^2 [1 - v_1] + \frac{1}{2} \sigma_v^2 Z^{s_{t+1}=0} + \frac{1}{2} \sigma_c^2 X^{s_{t+1}=0}, \quad (57)$$

and

$$r = \frac{p_{01}}{1 + k_{c,1}[p_{01} - p_{11}]} \quad (58)$$

$$1 - r = \frac{1 + k_{c,1}[p_{01} - p_{11}] - p_{01}}{1 + k_{c,1}[p_{01} - p_{11}]} \quad (59)$$

$$R = \frac{k_{c,1}[p_{01} - p_{11}] + p_{11}}{1 + k_{c,1}[p_{01} - p_{11}]} \quad (60)$$

$$1 - R = \frac{1 - p_{11}}{1 + k_{c,1}[p_{01} - p_{11}]} \quad (61)$$

A.2 The Price-Dividend Ratio

The coefficients governing the price-dividend ratio are found in an analogous manner as the coefficients for the price-consumption ratio above. The return on the market portfolio is again approximated as in Campbell and Shiller (1998) and the log price-dividend ratio again conjectured to be an affine function of our three state variables:

$$r_{m,t+1}(s_{t+1}) = k_{d,0} + k_{d,1} p d_{t+1}(s_{t+1}) - p d_t(s_t) + \Delta d_{t+1} \quad (62)$$

$$p d_t(s_t) = A_{d,0}(s_t) + A_{d,1}(s_t) x_{c,t} + A_{d,2}(s_t) x_{\pi,t} + A_{d,3}(s_t) \sigma_{\pi,t}^2, \quad (63)$$

with pd_t denoting the log price-dividend ratio. The constants k_d are functions of the average level of pd_t , which we denote by \bar{pd} .²²

The Euler condition for the market return is analogous to Equation ((38)):

$$0 = \sum_{s_{t+1}=0,1} p_{s_t, s_{t+1}} \{E[m_{t+1}(s_{t+1}) + r_{m,t+1}(s_{t+1})|I_{t+1}] + \frac{1}{2} \text{Var}[m_{t+1}(s_{t+1}) + r_{m,t+1}(s_{t+1})|I_{t+1}]\}, \quad (64)$$

with conditional mean and variance on the above expressions given by:

$$\begin{aligned} E[m_{t+1}(s_{t+1}) + r_{m,t+1}(s_{t+1})|I_{t+1}] &= \theta \ln(\delta) + \overbrace{\mu_c^{s_{t+1}} \left(\theta - \frac{\theta}{\psi} - 1 \right)}{=-\gamma} + (\theta - 1)(k_{c,0} - A_{c,0}^{s_t} + k_{c,1}\{A_{c,0}^{s_{t+1}} \\ &+ A_{c,3}^{s_{t+1}} \sigma_\pi^2 [1 - v_1]\}) + k_{d,0} + k_{d,1}\{A_{d,0}^{s_{t+1}} + A_{d,3}^{s_{t+1}} \sigma_\pi^2 [1 - v_1]\} - A_{d,0}^{s_t} + \mu_d^{s_{t+1}} \\ &+ x_{c,t}[\phi - \gamma + (\theta - 1)(k_{c,1}A_{c,1}^{s_{t+1}}\beta_1^{s_{t+1}} - A_{c,1}^{s_t}) + k_{d,1}A_{d,1}^{s_{t+1}}\beta_1^{s_{t+1}} - A_{d,1}^{s_t}] \\ &+ x_{\pi,t}[(\theta - 1)(k_{c,1}A_{c,1}^{s_{t+1}}\beta_2^{s_{t+1}} + k_{c,1}A_{c,2}^{s_{t+1}}\beta_4^{s_{t+1}} - A_{c,2}^{s_t}) + k_{d,1}A_{d,1}^{s_{t+1}}\beta_2^{s_{t+1}} \\ &+ k_{d,1}A_{d,2}^{s_{t+1}}\beta_4^{s_{t+1}} - A_{d,2}^{s_t}] \\ &+ \sigma_{\pi,t}^2[(\theta - 1)(k_{c,1}A_{c,3}^{s_{t+1}}v_1 - A_{c,3}^{s_t}) + k_{d,1}A_{d,3}^{s_{t+1}}v_1 - A_{d,3}^{s_t}] \end{aligned} \quad (65)$$

$$\text{Var}[m_{t+1}(s_{t+1}) + r_{m,t+1}(s_{t+1})|I_{t+1}] = \sigma_c^2 X^{s_{t+1}} + \sigma_{\pi,t}^2 Y^{s_{t+1}} + \sigma_v^2 Z^{s_{t+1}} \quad (66)$$

$$X^{s_{t+1}} = (\gamma^2 + \varphi^2 + \{(\theta - 1)k_{c,1}[A_{c,1}^{s_{t+1}}\delta_1^{s_{t+1}} + A_{c,2}^{s_{t+1}}\delta_3^{s_{t+1}}] + k_{d,1}[A_{d,1}^{s_{t+1}}\delta_1^{s_{t+1}} + A_{d,2}^{s_{t+1}}\delta_3^{s_{t+1}}]\})^2 \quad (67)$$

$$Y^{s_{t+1}} = [(\theta - 1)k_{c,1}(A_{c,1}^{s_{t+1}}\delta_2^{s_{t+1}} + A_{c,2}^{s_{t+1}}\delta_4^{s_{t+1}}) + k_{d,1}(A_{d,1}^{s_{t+1}}\delta_2^{s_{t+1}} + A_{d,2}^{s_{t+1}}\delta_4^{s_{t+1}})]^2 \quad (68)$$

$$Z^{s_{t+1}} = [(\theta - 1)k_{c,1}A_{c,3}^{s_{t+1}} + k_{d,1}A_{d,3}^{s_{t+1}}]^2 \quad (69)$$

²² $k_{d,1} = \frac{\exp(\bar{pd})}{1 + \exp(\bar{pd})}$ and $k_{d,0} = \ln(1 + \exp(\bar{pd})) - k_{d,1}\bar{pd}$.

Exploiting again the fact that the Euler Equation ((64)) must hold for both starting regimes $s_t = (0, 1)$ and for all values of our state variables allows us to solve for the A_d -coefficients:

$$A_{d,1}^0 = \left(\phi - \frac{1}{\psi}\right) \left[\frac{1 + \beta_1^1 k_{d,1} (p_{01} - p_{11})}{(1 - \beta_1^0 k_{d,1})(1 - p_{11} \beta_1^1 k_{d,1}) + p_{01} (\beta_1^0 k_{d,1} [1 - \beta_1^1 k_{d,1}])} \right] \quad (70)$$

$$A_{d,1}^1 = \left(\phi - \frac{1}{\psi}\right) \left[\frac{1 + \beta_1^0 k_{d,1} (p_{01} - p_{11})}{(1 - \beta_1^0 k_{d,1})(1 - p_{11} \beta_1^1 k_{d,1}) + p_{01} (\beta_1^0 k_{d,1} [1 - \beta_1^1 k_{d,1}])} \right] \quad (71)$$

$$A_{d,2}^0 = \frac{\beta_2^1 A_{d,1}^1 k_{d,1} p_{01} + \beta_2^0 A_{d,1}^0 k_{d,1} [p_{00} + \beta_4^1 k_{d,1} (p_{01} - p_{11})]}{(1 - \beta_4^0 k_{d,1}) + p_{01} [\beta_4^0 k_{d,1} (1 - \beta_4^1 k_{d,1})] + p_{11} [\beta_4^1 k_{d,1} (\beta_4^0 k_{d,1} - 1)]} \quad (72)$$

$$A_{d,2}^1 = \frac{\beta_2^0 A_{d,1}^0 k_{d,1} p_{10} + \beta_2^1 A_{d,1}^1 k_{d,1} [p_{11} + \beta_4^0 k_{d,1} (p_{01} - p_{11})]}{(1 - \beta_4^0 k_{d,1}) + p_{01} [\beta_4^0 k_{d,1} (1 - \beta_4^1 k_{d,1})] + p_{11} [\beta_4^1 k_{d,1} (\beta_4^0 k_{d,1} - 1)]} \quad (73)$$

$$A_{d,3}^0 = \frac{1}{1 - k_{d,1} v_1} [p_1 V_1 + p_2 V_2 + p_3 V_3 + p_4 V_4] \quad (74)$$

$$A_{d,3}^1 = \frac{1}{1 - k_{d,1} v_1} [P_1 V_1 + P_2 V_2 + P_3 V_3 + P_4 V_4] \quad (75)$$

$$V_1 = (\theta - 1)(k_{c,1} A_{c,3}^1 v_1 - A_{c,3}^0) + \frac{1}{2} Y^{s_{t+1}=1} \quad (76)$$

$$V_2 = (\theta - 1)(k_{c,1} A_{c,3}^0 v_1 - A_{c,3}^1) + \frac{1}{2} Y^{s_{t+1}=0} \quad (77)$$

$$V_3 = (\theta - 1)(k_{c,1} A_{c,3}^1 v_1 - A_{c,3}^1) + \frac{1}{2} Y^{s_{t+1}=1} \quad (78)$$

$$V_4 = (\theta - 1)(k_{c,1} A_{c,3}^0 v_1 - A_{c,3}^1) + \frac{1}{2} Y^{s_{t+1}=0} \quad (79)$$

$$1 = p_1 + p_2 + p_3 + p_4 \quad (80)$$

$$1 = P_1 + P_2 + P_3 + P_4, \quad (81)$$

with $Y^{s_{t+1}}$ as in (68) and the probabilities p_l and P_l given by:

$$p_1 = \frac{p_{01}(1 - k_{d,1}p_{11}v_1)}{1 + k_{d,1}v_1[p_{01} - p_{11}]} \quad (82)$$

$$p_2 = \frac{(1 - p_{01})(1 - k_{d,1}p_{11}v_1)}{1 + k_{d,1}v_1[p_{01} - p_{11}]} \quad (83)$$

$$p_3 = \frac{p_{11}p_{01}k_{d,1}v_1}{1 + k_{d,1}v_1[p_{01} - p_{11}]} \quad (84)$$

$$p_4 = \frac{(1 - p_{11})p_{01}k_{d,1}v_1}{1 + k_{d,1}v_1[p_{01} - p_{11}]} \quad (85)$$

$$P_1 = \frac{(1 - p_{11})p_{01}k_{d,1}v_1}{1 + k_{d,1}v_1[p_{01} - p_{11}]} \quad (86)$$

$$P_2 = \frac{(1 - p_{11})(1 - p_{01})k_{d,1}v_1}{1 + k_{d,1}v_1[p_{01} - p_{11}]} \quad (87)$$

$$P_3 = \frac{p_{11}(1 - k_{d,1}v_1[1 - p_{01}])}{1 + k_{d,1}v_1[p_{01} - p_{11}]} \quad (88)$$

$$P_4 = \frac{(1 - p_{11})(1 - k_{d,1}v_1[1 - p_{01}])}{1 + k_{d,1}v_1[p_{01} - p_{11}]} \quad (89)$$

$A_{d,0}^0$ and $A_{d,0}^1$ finally are given by:

$$A_{d,0}^0 = \frac{1}{1 - k_{d,1}} \{(1 - \theta)[r_1 A_{c,0}^1 + (1 - r_1)A_{c,0}^0] + r_2 V^{s_{t+1}=0} + (1 - r_2)V^{s_{t+1}=1}\} \quad (90)$$

$$A_{d,0}^1 = \frac{1}{1 - k_{d,1}} \{(1 - \theta)[R_1 A_{c,0}^1 + (1 - R_1)A_{c,0}^0] + R_2 V^{s_{t+1}=0} + (1 - R_2)V^{s_{t+1}=1}\}, \quad (91)$$

where

$$\begin{aligned} V^{s_{t+1}=0} &= \theta \ln(\delta) - \gamma \mu_c^0 + (\theta - 1)(k_{c,0} + k_{c,1}[A_{c,0}^0 + A_{c,3}^0 \sigma_\pi^2(1 - v_1)]) + k_{d,0} + k_{d,1}A_{d,3}^0 \sigma_\pi^2(1 - v_1) \\ &+ \mu_d + \frac{1}{2}\sigma_c^2 X^{s_{t+1}=0} + \frac{1}{2}\sigma_v^2 Z^{s_{t+1}=0} \end{aligned} \quad (92)$$

$$\begin{aligned} V^{s_{t+1}=1} &= \theta \ln(\delta) - \gamma \mu_c^1 + (\theta - 1)(k_{c,0} + k_{c,1}[A_{c,0}^1 + A_{c,3}^1 \sigma_\pi^2(1 - v_1)]) + k_{d,0} + k_{d,1}A_{d,3}^1 \sigma_\pi^2(1 - v_1) \\ &+ \mu_d + \frac{1}{2}\sigma_c^2 X^{s_{t+1}=1} + \frac{1}{2}\sigma_v^2 Z^{s_{t+1}=1}, \end{aligned} \quad (93)$$

with $X^{s_{t+1}}$ and $Z^{s_{t+1}}$ as in (67) and (69) and probabilities given by:

$$r_1 = \frac{k_{d,1}p_{01}}{1 + k_{d,1}[p_{01} - p_{11}]} \quad (94)$$

$$1 - r_1 = \frac{1 - k_{d,1}p_{11}}{1 + k_{d,1}[p_{01} - p_{11}]} \quad (95)$$

$$r_2 = \frac{1 + k_{d,1}[p_{01} - p_{11}] - p_{01}}{1 + k_{d,1}[p_{01} - p_{11}]} \quad (96)$$

$$1 - r_2 = \frac{p_{01}}{1 + k_{d,1}[p_{01} - p_{11}]} \quad (97)$$

$$R_1 = \frac{1 + k_{d,1}[p_{01} - 1]}{1 + k_{d,1}[p_{01} - p_{11}]} \quad (98)$$

$$1 - R_1 = \frac{k_{d,1}(1 - p_{11})}{1 + k_{d,1}[p_{01} - p_{11}]} \quad (99)$$

$$R_2 = \frac{1 - p_{11}}{1 + k_{d,1}[p_{01} - p_{11}]} \quad (100)$$

$$1 - R_2 = \frac{k_{d,1}[p_{01} - p_{11}] + p_{11}}{1 + k_{d,1}[p_{01} - p_{11}]} \quad (101)$$

A.3 Real Bonds

Let $y_{t,n} = -\frac{1}{n}q_{t,n}$ denote the n-period continuously compounded real yield with $q_{t,n}$ being the log price at time t of a real bond with maturity of n periods (t and n both are expressed in quarters). $q_{t,n}$ is conjectured to be a linear function of our three state variables:

$$q_{t,n}(s_t) = D_{0,n}(s_t) + D_{1,n}(s_t)x_{c,t} + D_{2,n}(s_t)x_{\pi,t} + D_{3,n}(s_t)\sigma_{\pi,t}^2 \quad (102)$$

The Euler equation for real bonds is:

$$1 = E[\exp\{m_{t+1}(s_{t+1}) + q_{t+1,n-1}(s_{t+1}) - q_{t,n}(s_t)\} | I_t] \quad (103)$$

Making use of the law of iterated expectations and of the conditional normality of log consump-

tion growth and the state variables and of the approximation $e^y - 1 \approx y$ analogous to Section A.1 and A.2, the Euler equation for real bonds can be rewritten as:

$$q_{t,n}(s_t) = \sum_{s_{t+1}=0,1} p_{s_t,s_{t+1}} \{E[m_{t+1}(s_{t+1}) + q_{t+1,n-1}(s_{t+1})|I_{t+1}] + \frac{1}{2}Var[m_{t+1}(s_{t+1}) + q_{t+1,n-1}(s_{t+1})|I_{t+1}]\} \quad (104)$$

The conditional mean and the conditional variance in the above expression (104) are given by:

$$\begin{aligned} E[m_{t+1}(s_{t+1}) + q_{t+1,n-1}(s_{t+1})|I_{t+1}] &= \theta \ln(\delta) + \overbrace{\mu_c^{s_{t+1}} (\theta - 1 - \frac{\theta}{\psi})}^{=-\gamma} + (\theta - 1)(k_{c,0} - A_{c,0}^{s_t} + k_{c,1}\{A_{c,0}^{s_{t+1}} \\ &+ A_{c,3}^{s_{t+1}}\sigma_\pi^2(1 - v_1)\}) + D_{0,n-1}^{s_{t+1}} + D_{3,n-1}^{s_{t+1}}\sigma_\pi^2(1 - v_1) \\ &+ x_{c,t}[-\gamma + (\theta - 1)(k_{c,1}A_{c,1}^{s_{t+1}}\beta_1^{s_{t+1}} - A_{c,1}^{s_t}) + D_{1,n-1}^{s_{t+1}}\beta_1^{s_{t+1}}] \\ &+ x_{\pi,t}[(\theta - 1)(k_{c,1}A_{c,1}^{s_{t+1}}\beta_2^{s_{t+1}} + k_{c,1}A_{c,2}^{s_{t+1}}\beta_4^{s_{t+1}} - A_{c,2}^{s_t}) + D_{1,n-1}^{s_{t+1}}\beta_2^{s_{t+1}} \\ &+ D_{2,n-1}^{s_{t+1}}\beta_4^{s_{t+1}}] \\ &+ \sigma_{\pi,t}^2[(\theta - 1)(k_{c,1}A_{c,3}^{s_{t+1}}v_1 - A_{c,3}^{s_t}) + D_{3,n-1}^{s_{t+1}}v_1] \end{aligned} \quad (105)$$

$$Var[m_{t+1}(s_{t+1}) + q_{t+1,n-1}(s_{t+1})|I_{t+1}] = \sigma_c^2 X^{s_{t+1}} + \sigma_{\pi,t}^2 Y^{s_{t+1}} + \sigma_v^2 Z^{s_{t+1}} \quad (106)$$

$$X^{s_{t+1}} = \gamma^2 + [(\theta - 1)(k_{c,1}A_{c,1}^{s_{t+1}}\delta_1^{s_{t+1}} + k_{c,1}A_{c,2}^{s_{t+1}}\delta_3^{s_{t+1}}) + D_{1,n-1}^{s_{t+1}}\delta_1^{s_{t+1}} + D_{2,n-1}^{s_{t+1}}\delta_3^{s_{t+1}}]^2 \quad (107)$$

$$Y^{s_{t+1}} = [(\theta - 1)(k_{c,1}A_{c,1}^{s_{t+1}}\delta_2^{s_{t+1}} + k_{c,1}A_{c,2}^{s_{t+1}}\delta_4^{s_{t+1}}) + D_{1,n-1}^{s_{t+1}}\delta_2^{s_{t+1}} + D_{2,n-1}^{s_{t+1}}\delta_4^{s_{t+1}}]^2 \quad (108)$$

$$Z^{s_{t+1}} = [(\theta - 1)k_{c,1}A_{c,3}^{s_{t+1}} + D_{3,n-1}^{s_{t+1}}]^2. \quad (109)$$

Exploiting the fact that Equation ((104)) must hold for both starting regimes $s_t = (0, 1)$ and for all values of our state variables and that $D_{i,0} = 0$ for $i = 0, 1, 2, 3$, allows us to solve for the

D-coefficients:

$$D_{1,n}^0 = -\frac{1}{\psi} + p_{00}D_{1,n-1}^0\beta_1^0 + p_{01}D_{1,n-1}^1\beta_1^1 \quad (110)$$

$$D_{1,n}^1 = -\frac{1}{\psi} + p_{10}D_{1,n-1}^0\beta_1^0 + p_{11}D_{1,n-1}^1\beta_1^1 \quad (111)$$

$$D_{2,n}^0 = p_{00}[D_{1,n-1}^0\beta_2^0 + D_{2,n-1}^0\beta_4^0] + p_{01}[D_{1,n-1}^1\beta_2^1 + D_{2,n-1}^1\beta_4^1] \quad (112)$$

$$D_{2,n}^1 = p_{11}[D_{1,n-1}^1\beta_2^1 + D_{2,n-1}^1\beta_4^1] + p_{10}[D_{1,n-1}^0\beta_2^0 + D_{2,n-1}^0\beta_4^0] \quad (113)$$

$$\begin{aligned} D_{3,n}^0 &= p_{00}[(\theta - 1)A_{c,3}^0(k_{c,1}v_1 - 1) + D_{3,n-1}^0v_1 + \frac{1}{2}Y^{s_{t+1}=0}] \\ &+ p_{01}[(\theta - 1)(k_{c,1}v_1A_{c,3}^1 - A_{c,3}^0) + D_{3,n-1}^1v_1 + \frac{1}{2}Y^{s_{t+1}=1}] \end{aligned} \quad (114)$$

$$\begin{aligned} D_{3,n}^1 &= p_{11}[(\theta - 1)A_{c,3}^1(k_{c,1}v_1 - 1) + D_{3,n-1}^1v_1 + \frac{1}{2}Y^{s_{t+1}=1}] \\ &+ p_{10}[(\theta - 1)(k_{c,1}v_1A_{c,3}^0 - A_{c,3}^1) + D_{3,n-1}^0v_1 + \frac{1}{2}Y^{s_{t+1}=0}] \end{aligned} \quad (115)$$

$D_{0,n}^0$ and $D_{0,n}^1$ finally are given by:

$$\begin{aligned} D_{0,n}^0 &= p_{00}[\theta \ln(\delta) - \gamma\mu_c^0 + (\theta - 1)(k_{c,0} - A_{c,0}^0 + k_{c,1}A_{c,0}^0 + k_{c,1}A_{c,3}^0\sigma_\pi^2(1 - v_1)) + D_{0,n-1}^0 \\ &+ D_{3,n-1}^0\sigma_\pi^2(1 - v_1) + \frac{1}{2}(\sigma_v^2Z^{s_{t+1}=0} + \sigma_c^2X^{s_{t+1}=0})] \\ &+ p_{01}[\theta \ln(\delta) - \gamma\mu_c^1 + (\theta - 1)(k_{c,0} - A_{c,0}^0 + k_{c,1}A_{c,0}^1 + k_{c,1}A_{c,3}^1\sigma_\pi^2(1 - v_1)) + D_{0,n-1}^1 \\ &+ D_{3,n-1}^1\sigma_\pi^2(1 - v_1) + \frac{1}{2}(\sigma_v^2Z^{s_{t+1}=1} + \sigma_c^2X^{s_{t+1}=1})] \end{aligned} \quad (116)$$

$$\begin{aligned} D_{0,n}^1 &= p_{11}[\theta \ln(\delta) - \gamma\mu_c^1 + (\theta - 1)(k_{c,0} - A_{c,0}^1 + k_{c,1}A_{c,0}^1 + k_{c,1}A_{c,3}^1\sigma_\pi^2(1 - v_1)) + D_{0,n-1}^1 \\ &+ D_{3,n-1}^1\sigma_\pi^2(1 - v_1) + \frac{1}{2}(\sigma_v^2Z^{s_{t+1}=1} + \sigma_c^2X^{s_{t+1}=1})] \\ &+ p_{10}[\theta \ln(\delta) - \gamma\mu_c^0 + (\theta - 1)(k_{c,0} - A_{c,0}^1 + k_{c,1}A_{c,0}^0 + k_{c,1}A_{c,3}^0\sigma_\pi^2(1 - v_1)) + D_{0,n-1}^0 \\ &+ D_{3,n-1}^0\sigma_\pi^2(1 - v_1) + \frac{1}{2}(\sigma_v^2Z^{s_{t+1}=0} + \sigma_c^2X^{s_{t+1}=0})] \end{aligned} \quad (117)$$

A.4 Nominal Bonds

Let $y_{t,n}^{\$} = -\frac{1}{n}q_{t,n}^{\$}$ denote the n-period continuously compounded nominal yield with $q_{t,n}^{\$}$ being the log price at time t of a nominal bond with maturity of n periods. $q_{t,n}^{\$}$ is conjectured to be a linear function of our state variables:

$$q_{t,n}^{\$}(s_t) = D_{0,n}^{\$}(s_t) + D_{1,n}^{\$}(s_t)x_{c,t} + D_{2,n}^{\$}(s_t)x_{\pi,t} + D_{3,n}^{\$}(s_t)\sigma_{\pi,t}^2 \quad (118)$$

The Euler equation for nominal bonds is:

$$1 = E[\exp\{m_{t+1}(s_{t+1}) + q_{t+1,n-1}^{\$}(s_{t+1}) - q_{t,n}^{\$}(s_t) - \pi_{t+1}\}|I_t] \quad (119)$$

Making use of the law of iterated expectations, the conditional normality of log consumption growth and the state variables and of the approximation $e^y - 1 \approx y$ analogous to Section A.1 and A.2, the Euler equation for nominal bonds can be rewritten as:

$$\begin{aligned} q_{t,n}^{\$}(s_t) &= \sum_{s_{t+1}=0,1} p_{s_t,s_{t+1}} \{E[m_{t+1}(s_{t+1}) - \pi_{t+1} + q_{t+1,n-1}^{\$}(s_{t+1})|I_{t+1}] \\ &\quad + \frac{1}{2}Var[m_{t+1}(s_{t+1}) - \pi_{t+1} + q_{t+1,n-1}^{\$}(s_{t+1})|I_{t+1}]\} \end{aligned} \quad (120)$$

The conditional mean and the conditional variance in the above expression (120) are given by:

$$\begin{aligned}
E[m_{t+1}(s_{t+1}) - \pi_{t+1} + q_{t+1,n-1}^{\mathbb{S}}(s_{t+1})|I_{t+1}] &= [\theta \ln(\delta) - \mu_c^{s_{t+1}}\gamma + (\theta - 1)(k_{c,0} - A_{c,0}^{s_t} + k_{c,1}\{A_{c,0}^{s_{t+1}} \\
&+ A_{c,3}^{s_{t+1}}\sigma_{\pi}^2(1 - v_1)\}) + D_{0,n-1}^{\mathbb{S}^{s_{t+1}}} + D_{3,n-1}^{\mathbb{S}^{s_{t+1}}}\sigma_{\pi}^2(1 - v_1) - \mu_{\pi}^{s_{t+1}}] \\
&+ x_{c,t}[-\gamma + (\theta - 1)(k_{c,1}A_{c,1}^{s_{t+1}}\beta_1^{s_{t+1}} - A_{c,1}^{s_t}) + D_{1,n-1}^{\mathbb{S}^{s_{t+1}}}\beta_1^{s_{t+1}}] \\
&+ x_{\pi,t}[(\theta - 1)(k_{c,1}A_{c,1}^{s_{t+1}}\beta_2^{s_{t+1}} + k_{c,1}A_{c,2}^{s_{t+1}}\beta_4^{s_{t+1}} - A_{c,2}^{s_t}) \\
&+ D_{1,n-1}^{\mathbb{S}^{s_{t+1}}}\beta_2^{s_{t+1}} + D_{2,n-1}^{\mathbb{S}^{s_{t+1}}}\beta_4^{s_{t+1}} - 1] \\
&+ \sigma_{\pi,t}^2[(\theta - 1)(k_{c,1}A_{c,3}^{s_{t+1}}v_1 - A_{c,3}^{s_t}) + D_{3,n-1}^{\mathbb{S}^{s_{t+1}}}v_1] \quad (121)
\end{aligned}$$

$$\text{Var}[m_{t+1}(s_{t+1}) - \pi_{t+1} + q_{t+1,n-1}^{\mathbb{S}}(s_{t+1})|I_{t+1}] = \sigma_c^2 X^{s_{t+1}} + \sigma_{\pi,t}^2 Y^{s_{t+1}} + \sigma_v^2 Z^{s_{t+1}} \quad (122)$$

$$X^{s_{t+1}} = \gamma^2 + [(\theta - 1)(k_{c,1}A_{c,1}^{s_{t+1}}\delta_1^{s_{t+1}} + k_{c,1}A_{c,2}^{s_{t+1}}\delta_3^{s_{t+1}}) + D_{1,n-1}^{\mathbb{S}^{s_{t+1}}}\delta_1^{s_{t+1}} + D_{2,n-1}^{\mathbb{S}^{s_{t+1}}}\delta_3^{s_{t+1}}]^2 \quad (123)$$

$$Y^{s_{t+1}} = [(\theta - 1)(k_{c,1}A_{c,1}^{s_{t+1}}\delta_2^{s_{t+1}} + k_{c,1}A_{c,2}^{s_{t+1}}\delta_4^{s_{t+1}}) + D_{1,n-1}^{\mathbb{S}^{s_{t+1}}}\delta_2^{s_{t+1}} + D_{2,n-1}^{\mathbb{S}^{s_{t+1}}}\delta_4^{s_{t+1}}]^2 + 1 \quad (124)$$

$$Z^{s_{t+1}} = [(\theta - 1)k_{c,1}A_{c,3}^{s_{t+1}} + D_{3,n-1}^{\mathbb{S}^{s_{t+1}}}]^2. \quad (125)$$

Exploiting the fact that Equation ((120)) must hold for both starting regimes $s_t = (0, 1)$ and for all values of our state variables and that $D_{i,0}^{\mathbb{S}} = 0$ for $i = 0, 1, 2, 3$ allows us to solve a system of 8 equations for the 8 unknown $D^{\mathbb{S}}$ -coefficients:

$$D_{1,n}^{\$0} = -\frac{1}{\psi} + p_{00}D_{1,n-1}^{\$0}\beta_1^0 + p_{01}D_{1,n-1}^{\$1}\beta_1^1 \quad (126)$$

$$D_{1,n}^{\$1} = -\frac{1}{\psi} + p_{10}D_{1,n-1}^{\$0}\beta_1^0 + p_{11}D_{1,n-1}^{\$1}\beta_1^1 \quad (127)$$

$$D_{2,n}^{\$0} = p_{00}[D_{1,n-1}^{\$0}\beta_2^0 + D_{2,n-1}^{\$0}\beta_4^0] + p_{01}[D_{1,n-1}^{\$1}\beta_2^1 + D_{2,n-1}^{\$1}\beta_4^1] - 1 \quad (128)$$

$$D_{2,n}^{\$1} = p_{11}[D_{1,n-1}^{\$1}\beta_2^1 + D_{2,n-1}^{\$1}\beta_4^1] + p_{10}[D_{1,n-1}^{\$0}\beta_2^0 + D_{2,n-1}^{\$0}\beta_4^0] - 1 \quad (129)$$

$$\begin{aligned} D_{3,n}^{\$0} &= p_{00}[(\theta - 1)(A_{c,3}^0 k_{c,1} v_1 - A_{c,3}^0) + D_{3,n-1}^{\$0} v_1 + \frac{1}{2} Y^{st+1=0}] \\ &+ p_{01}[(\theta - 1)(k_{c,1} v_1 A_{c,3}^1 - A_{c,3}^0) + D_{3,n-1}^{\$1} v_1 + \frac{1}{2} Y^{st+1=1}] \end{aligned} \quad (130)$$

$$\begin{aligned} D_{3,n}^{\$1} &= p_{11}[(\theta - 1)(A_{c,3}^1 k_{c,1} v_1 - A_{c,3}^1) + D_{3,n-1}^{\$1} v_1 + \frac{1}{2} Y^{st+1=1}] \\ &+ p_{10}[(\theta - 1)(k_{c,1} v_1 A_{c,3}^0 - A_{c,3}^1) + D_{3,n-1}^{\$0} v_1 + \frac{1}{2} Y^{st+1=0}] \end{aligned} \quad (131)$$

$D_{0,n}^{\$0}$ and $D_{0,n}^{\$1}$ finally are given by:

$$\begin{aligned} D_{0,n}^{\$0} &= p_{00}[(\theta \ln(\delta) - \gamma \mu_c^0 + (\theta - 1)(k_{c,0} - A_{c,0}^0 + k_{c,1} A_{c,0}^0 + k_{c,1} A_{c,3}^0 \sigma_\pi^2 (1 - v_1)) + D_{0,n-1}^{\$0} \\ &+ D_{3,n-1}^{\$0} \sigma_\pi^2 (1 - v_1) - \mu_\pi^0 + \frac{1}{2} (\sigma_v^2 Z^{st+1=0} + \sigma_c^2 X^{st+1=0})] \\ &+ p_{01}[(\theta \ln(\delta) - \gamma \mu_c^1 + (\theta - 1)(k_{c,0} - A_{c,0}^0 + k_{c,1} A_{c,0}^1 + k_{c,1} A_{c,3}^1 \sigma_\pi^2 (1 - v_1)) + D_{0,n-1}^{\$1} \\ &+ D_{3,n-1}^{\$1} \sigma_\pi^2 (1 - v_1) - \mu_\pi^1 + \frac{1}{2} (\sigma_v^2 Z^{st+1=1} + \sigma_c^2 X^{st+1=1})] \end{aligned} \quad (132)$$

$$\begin{aligned} D_{0,n}^{\$1} &= p_{11}[(\theta \ln(\delta) - \gamma \mu_c^1 + (\theta - 1)(k_{c,0} - A_{c,0}^1 + k_{c,1} A_{c,0}^1 + k_{c,1} A_{c,3}^1 \sigma_\pi^2 (1 - v_1)) + D_{0,n-1}^{\$1} \\ &+ D_{3,n-1}^{\$1} \sigma_\pi^2 (1 - v_1) - \mu_\pi^1 + \frac{1}{2} (\sigma_v^2 Z^{st+1=1} + \sigma_c^2 X^{st+1=1})] \\ &+ p_{10}[(\theta \ln(\delta) - \gamma \mu_c^0 + (\theta - 1)(k_{c,0} - A_{c,0}^1 + k_{c,1} A_{c,0}^0 + k_{c,1} A_{c,3}^0 \sigma_\pi^2 (1 - v_1)) + D_{0,n-1}^{\$0} \\ &+ D_{3,n-1}^{\$0} \sigma_\pi^2 (1 - v_1) - \mu_\pi^0 + \frac{1}{2} (\sigma_v^2 Z^{st+1=0} + \sigma_c^2 X^{st+1=0})] \end{aligned} \quad (133)$$

Innovations and Analytical Risk Premia:

In the following Sections B.1 - B.4 we show the derivation of analytical risk premia for stocks as well as real and nominal bonds:

B.1 Risk Premium Formula

The conditional risk premium (given I_t) for any asset i can be derived using the Euler condition for this asset i together with the Euler equation for the 1-period risk-free rate. It is straightforward to show that the risk premium for any asset i can be expressed as:

$$\begin{aligned} E[r_{i,t+1}(s_{t+1}) - r_{f,t}|I_t] &+ \frac{1}{2} \sum_{s_{t+1}=0,1} p_{s_t,s_{t+1}} \text{Var}[r_{i,t+1}(s_{t+1})|I_{t+1}] \\ &= - \sum_{s_{t+1}=0,1} p_{s_t,s_{t+1}} \text{Cov}[m_{t+1}(s_{t+1}), r_{i,t+1}(s_{t+1})|I_{t+1}] \end{aligned} \quad (134)$$

This equation holds for both current states $s_t = (0, 1)$. Hence, given the innovations to the real pricing kernel and to the assets under considerations, we can easily derive analytical RP expressions.

B.2 Innovations

This section shows the innovations to stock and bond returns, nominal yields and price-dividend ratios.

The following expression represents innovations to the real pricing kernel, with λ' s representing the regime-dependent market prices of risk:

$$m_{t+1}(s_{t+1}) - E[m_{t+1}(s_{t+1})|I_{t+1}] = -\lambda_{\eta_c} \sigma_c \eta_{c,t+1} - \lambda_{\varepsilon_c}(s_{t+1}) \sigma_c \varepsilon_{c,t+1} - \lambda_\nu(s_{t+1}) \sigma_\nu w_{t+1} - \lambda_{\varepsilon_\pi}(s_{t+1}) \sigma_{\pi,t} \varepsilon_{\pi,t+1} \quad (135)$$

$$\lambda_{\eta_c} = \gamma \quad (136)$$

$$\lambda_{\varepsilon_c}(s_{t+1}) = (1 - \theta)[k_{c,1} A_{c,1}(s_{t+1}) \delta_1(s_{t+1}) + k_{c,1} A_{c,2}(s_{t+1}) \delta_3(s_{t+1})] \quad (137)$$

$$\lambda_\nu(s_{t+1}) = (1 - \theta) k_{c,1} A_{c,3}(s_{t+1}) \quad (138)$$

$$\lambda_{\varepsilon_\pi}(s_{t+1}) = (1 - \theta)[k_{c,1} A_{c,1}(s_{t+1}) \delta_2(s_{t+1}) + k_{c,1} A_{c,2}(s_{t+1}) \delta_4(s_{t+1})] \quad (139)$$

$$m_{t+1}(s_{t+1}) - E[m_{t+1}(s_{t+1})|I_t] = m_{t+1}(s_{t+1}) - E[m_{t+1}(s_{t+1})|I_{t+1}] + V(s_t, s_{t+1}) \quad (140)$$

$$\begin{aligned} V(s_t, s_{t+1}) &= -\gamma[\mu_c(s_{t+1}) - p_{s_t,0} \mu_c^0 - p_{s_t,1} \mu_c^1] + (\theta - 1) k_{c,1} [A_{c,0}(s_{t+1}) - p_{s_t,0} A_{c,0}^0 - p_{s_t,1} A_{c,0}^1] \\ &+ (\theta - 1) k_{c,1} x_{c,t} [A_{c,1}(s_{t+1}) \beta_1(s_{t+1}) - p_{s_t,0} A_{c,1}^0 \beta_1^0 - p_{s_t,1} A_{c,1}^1 \beta_1^1] \\ &+ (\theta - 1) k_{c,1} x_{\pi,t} [A_{c,1}(s_{t+1}) \beta_2(s_{t+1}) - p_{s_t,0} A_{c,1}^0 \beta_2^0 - p_{s_t,1} A_{c,1}^1 \beta_2^1] \\ &+ A_{c,2}(s_{t+1}) \beta_4(s_{t+1}) - p_{s_t,0} A_{c,2}^0 \beta_4^0 - p_{s_t,1} A_{c,2}^1 \beta_4^1 \\ &+ (\theta - 1) k_{c,1} (\sigma_\pi^2 (1 - v_1) + v_1 \sigma_{\pi,t}^2) [A_{c,3}(s_{t+1}) - p_{s_t,0} A_{c,3}^0 - p_{s_t,1} A_{c,3}^1] \end{aligned} \quad (141)$$

The innovations to the (real) return of the market portfolio:

$$\begin{aligned} r_{m,t+1}(s_{t+1}) - E[r_{m,t+1}(s_{t+1})|I_{t+1}] &= \sigma_c \varepsilon_{c,t+1} [k_{d,1} A_{d,1}(s_{t+1}) \delta_1(s_{t+1}) + k_{d,1} A_{d,2}(s_{t+1}) \delta_3(s_{t+1})] \\ &+ \sigma_{\pi,t} \varepsilon_{\pi,t+1} [k_{d,1} A_{d,1}(s_{t+1}) \delta_2(s_{t+1}) + k_{d,1} A_{d,2}(s_{t+1}) \delta_4(s_{t+1})] \\ &+ \sigma_\nu w_{t+1} k_{d,1} A_{d,3}(s_{t+1}) + \varphi \sigma_c \eta_{d,t+1} \end{aligned} \quad (142)$$

$$r_{m,t+1}(s_{t+1}) - E[r_{m,t+1}(s_{t+1})|I_t] = r_{m,t+1}(s_{t+1}) - E[r_{m,t+1}(s_{t+1})|I_{t+1}] + R(s_t, s_{t+1}) \quad (143)$$

$$\begin{aligned} R(s_t, s_{t+1}) &= [\mu_d(s_{t+1}) - p_{s_t,0}\mu_d^0 - p_{s_t,1}\mu_d^1] + k_{d,1}[A_{d,0}(s_{t+1}) - p_{s_t,0}A_{d,0}^0 - p_{s_t,1}A_{d,0}^1] \\ &+ k_{d,1}x_{c,t}[A_{d,1}(s_{t+1})\beta_1(s_{t+1}) - p_{s_t,0}A_{d,1}^0\beta_1^0 - p_{s_t,1}A_{d,1}^1\beta_1^1] \\ &+ k_{d,1}x_{\pi,t}[A_{d,1}(s_{t+1})\beta_2(s_{t+1}) - p_{s_t,0}A_{d,1}^0\beta_2^0 - p_{s_t,1}A_{d,1}^1\beta_2^1] \\ &+ A_{d,2}(s_{t+1})\beta_4(s_{t+1}) - p_{s_t,0}A_{d,2}^0\beta_4^0 - p_{s_t,1}A_{d,2}^1\beta_4^1] \\ &+ (\sigma_\pi^2(1 - v_1) + v_1\sigma_{\pi,t}^2)[A_{d,3}(s_{t+1}) - p_{s_t,0}A_{d,3}^0 - p_{s_t,1}A_{d,3}^1] \end{aligned} \quad (144)$$

The return from holding a n-period real bond for one period is $h_{t+1,n}(s_{t+1}) = q_{t+1,n-1}(s_{t+1}) - q_{t,n}(s_t)$. The innovation to this return is ²³:

$$\begin{aligned} h_{t+1,n}(s_{t+1}) - E[h_{t+1,n}(s_{t+1})|I_{t+1}] &= \sigma_c \varepsilon_{c,t+1} [D_{1,n-1}(s_{t+1})\delta_1(s_{t+1}) + D_{2,n-1}(s_{t+1})\delta_3(s_{t+1})] \\ &+ \sigma_{\pi,t} \varepsilon_{\pi,t+1} [D_{1,n-1}(s_{t+1})\delta_2(s_{t+1}) + D_{2,n-1}(s_{t+1})\delta_4(s_{t+1})] \\ &+ \sigma_v w_{t+1} D_{3,n-1}(s_{t+1}) \end{aligned} \quad (145)$$

$$h_{t+1,n}(s_{t+1}) - E[h_{t+1,n}(s_{t+1})|I_t] = h_{t+1,n}(s_{t+1}) - E[h_{t+1,n}(s_{t+1})|I_{t+1}] + H(s_t, s_{t+1}) \quad (146)$$

²³To get the innovation to the return of a nominal bond, just replace the D-coefficients by $D^{\$}$ -coefficients.

$$\begin{aligned}
H(s_t, s_{t+1}) &= [D_{0,n-1}(s_{t+1}) - p_{s_t,0}D_{0,n-1}^0 - p_{s_t,1}D_{0,n-1}^1] \\
&+ x_{c,t}[D_{1,n-1}(s_{t+1})\beta_1(s_{t+1}) - p_{s_t,0}D_{1,n-1}^0\beta_1^0 - p_{s_t,1}D_{1,n-1}^1\beta_1^1] \\
&+ x_{\pi,t}[D_{1,n-1}(s_{t+1})\beta_2(s_{t+1}) - p_{s_t,0}D_{1,n-1}^0\beta_2^0 - p_{s_t,1}D_{1,n-1}^1\beta_2^1] \\
&+ D_{2,n-1}(s_{t+1})\beta_4(s_{t+1}) - p_{s_t,0}D_{2,n-1}^0\beta_4^0 - p_{s_t,1}D_{2,n-1}^1\beta_4^1] \\
&+ (\sigma_\pi^2(1 - v_1) + v_1\sigma_{\pi,t}^2)[D_{3,n-1}(s_{t+1}) - p_{s_t,0}D_{3,n-1}^0 - p_{s_t,1}D_{3,n-1}^1] \quad (147)
\end{aligned}$$

Innovations to nominal yields:

$$\begin{aligned}
y_{t+1,n}^\$(s_{t+1}) - E[y_{t+1,n}^\$(s_{t+1})|I_{t+1}] &= -\frac{1}{n} \left[\sigma_c \varepsilon_{c,t+1} [D_{1,n-1}^\$(s_{t+1})\delta_1(s_{t+1}) + D_{2,n-1}^\$(s_{t+1})\delta_3(s_{t+1})] \right. \\
&+ \sigma_\pi \varepsilon_{\pi,t+1} [D_{1,n-1}^\$(s_{t+1})\delta_2(s_{t+1}) + D_{2,n-1}^\$(s_{t+1})\delta_4(s_{t+1})] \\
&\left. + \sigma_v w_{t+1} D_{3,n-1}^\$(s_{t+1}) \right] \quad (148)
\end{aligned}$$

$$y_{t+1,n}^\$(s_{t+1}) - E[y_{t+1,n}^\$(s_{t+1})|I_t] = y_{t+1,n}^\$(s_{t+1}) - E[y_{t+1,n}^\$(s_{t+1})|I_{t+1}] + Q(s_t, s_{t+1}) \quad (149)$$

$$\begin{aligned}
Q(s_t, s_{t+1}) &= -\frac{1}{n} \left[[D_{0,n}^\$(s_{t+1}) - p_{s_t,0}D_{0,n}^{\$,0} - p_{s_t,1}D_{0,n}^{\$,1}] \right. \\
&+ x_{c,t}[D_{1,n}^\$(s_{t+1})\beta_1(s_{t+1}) - p_{s_t,0}D_{1,n}^{\$,0}\beta_1^0 - p_{s_t,1}D_{1,n}^{\$,1}\beta_1^1] \\
&+ x_{\pi,t}[D_{1,n}^\$(s_{t+1})\beta_2(s_{t+1}) - p_{s_t,0}D_{1,n}^{\$,0}\beta_2^0 - p_{s_t,1}D_{1,n}^{\$,1}\beta_2^1] \\
&+ D_{2,n}^\$(s_{t+1})\beta_4(s_{t+1}) - p_{s_t,0}D_{2,n}^{\$,0}\beta_4^0 - p_{s_t,1}D_{2,n}^{\$,1}\beta_4^1] \\
&\left. + (\sigma_\pi^2(1 - v_1) + v_1\sigma_{\pi,t}^2)[D_{3,n}^\$(s_{t+1}) - p_{s_t,0}D_{3,n}^{\$,0} - p_{s_t,1}D_{3,n}^{\$,1}] \right] \quad (150)
\end{aligned}$$

And finally the innovations to the price-dividend ratio:

$$\begin{aligned}
pd_{t+1}(s_{t+1}) - E[pd_{t+1}(s_{t+1})|I_{t+1}] &= \sigma_c \varepsilon_{c,t+1} [A_{d,1}(s_{t+1})\delta_1(s_{t+1}) + A_{d,2}(s_{t+1})\delta_3(s_{t+1})] \\
&+ \sigma_{\pi,t} \varepsilon_{\pi,t+1} [A_{d,1}(s_{t+1})\delta_2(s_{t+1}) + A_{d,2}(s_{t+1})\delta_4(s_{t+1})] \\
&+ \sigma_v w_{t+1} A_{d,3}(s_{t+1})
\end{aligned} \tag{151}$$

$$pd_{t+1}(s_{t+1}) - E[pd_{t+1}(s_{t+1})|I_t] = pd_{t+1}(s_{t+1}) - E[pd_{t+1}(s_{t+1})|I_{t+1}] + S(s_t, s_{t+1}) \tag{152}$$

$$\begin{aligned}
S(s_t, s_{t+1}) &= [A_{d,0}(s_{t+1}) - p_{s_t,0} A_{d,0}^0 - p_{s_t,1} A_{d,0}^1] \\
&+ x_{c,t} [A_{d,1}(s_{t+1})\beta_1(s_{t+1}) - p_{s_t,0} A_{d,1}^0 \beta_1^0 - p_{s_t,1} A_{d,1}^1 \beta_1^1] \\
&+ x_{\pi,t} [A_{d,1}(s_{t+1})\beta_2(s_{t+1}) - p_{s_t,0} A_{d,1}^0 \beta_2^0 - p_{s_t,1} A_{d,1}^1 \beta_2^1] \\
&+ A_{d,2}(s_{t+1})\beta_4(s_{t+1}) - p_{s_t,0} A_{d,2}^0 \beta_4^0 - p_{s_t,1} A_{d,2}^1 \beta_4^1] \\
&+ (\sigma_{\pi}^2(1 - v_1) + v_1 \sigma_{\pi,t}^2) [A_{d,3}(s_{t+1}) - p_{s_t,0} A_{d,3}^0 - p_{s_t,1} A_{d,3}^1]
\end{aligned} \tag{153}$$

B.3 Risk Premia

Using the formula from B.1, the risk premium for the market portfolio can be expressed as:

$$\begin{aligned}
E[r_{m,t+1}(s_{t+1}) - r_{f,t}|I_t] &+ \frac{1}{2} \sum_{s_{t+1}=0,1} p_{s_t,s_{t+1}} \text{Var}[r_{m,t+1}(s_{t+1})|I_{t+1}] \\
&= \sum_{s_{t+1}=0,1} p_{s_t,s_{t+1}} [A(s_{t+1}) + B(s_{t+1})\sigma_{\pi,t}^2]
\end{aligned} \tag{154}$$

$$A(s_{t+1}) = \sigma_c^2 [k_{d,1} A_{d,1}(s_{t+1}) \delta_1(s_{t+1}) + k_{d,1} A_{d,2}(s_{t+1}) \delta_3(s_{t+1})] \lambda_{\varepsilon_c}(s_{t+1}) + \sigma_v^2 k_{d,1} A_{d,3}(s_{t+1}) \lambda_v(s_{t+1}) \quad (155)$$

$$B(s_{t+1}) = [k_{d,1} A_{d,1}(s_{t+1}) \delta_2(s_{t+1}) + k_{d,1} A_{d,2}(s_{t+1}) \delta_4(s_{t+1})] \lambda_{\varepsilon_\pi}(s_{t+1}) \quad (156)$$

The risk premium for real bonds is:

$$\begin{aligned} E[h_{t+1,n}(s_{t+1}) - r_{f,t}|I_t] &+ \frac{1}{2} \sum_{s_{t+1}=0,1} p_{s_t,s_{t+1}} \text{Var}[h_{t+1,n}(s_{t+1})|I_{t+1}] \\ &= \sum_{s_{t+1}=0,1} p_{s_t,s_{t+1}} [A(s_{t+1}) + B(s_{t+1}) \sigma_{\pi,t}^2] \end{aligned} \quad (157)$$

$$A(s_{t+1}) = \sigma_c^2 [D_{1,n-1}(s_{t+1}) \delta_1(s_{t+1}) + D_{2,n-1}(s_{t+1}) \delta_3(s_{t+1})] \lambda_{\varepsilon_c}(s_{t+1}) + \sigma_v^2 D_{3,n-1}(s_{t+1}) \lambda_v(s_{t+1}) \quad (158)$$

$$B(s_{t+1}) = [D_{1,n-1}(s_{t+1}) \delta_2(s_{t+1}) + D_{2,n-1}(s_{t+1}) \delta_4(s_{t+1})] \lambda_{\varepsilon_\pi}(s_{t+1}) \quad (159)$$

The risk premium for nominal bonds:

$$\begin{aligned} E[h_{t+1,n}^\$(s_{t+1}) - r_{f,t}^\$|I_t] &+ \frac{1}{2} \sum_{s_{t+1}=0,1} p_{s_t,s_{t+1}} \text{Var}[h_{t+1,n}^\$(s_{t+1})|I_{t+1}] \\ &= \sum_{s_{t+1}=0,1} p_{s_t,s_{t+1}} [A(s_{t+1}) + B(s_{t+1}) \sigma_{\pi,t}^2] \end{aligned} \quad (160)$$

$$A(s_{t+1}) = \sigma_c^2 [D_{1,n-1}^\$(s_{t+1}) \delta_1(s_{t+1}) + D_{2,n-1}^\$(s_{t+1}) \delta_3(s_{t+1})] \lambda_{\varepsilon_c}(s_{t+1}) + \sigma_v^2 D_{3,n-1}^\$(s_{t+1}) \lambda_v(s_{t+1}) \quad (161)$$

$$B(s_{t+1}) = [D_{1,n-1}^\$(s_{t+1}) \delta_2(s_{t+1}) + D_{2,n-1}^\$(s_{t+1}) \delta_4(s_{t+1})] \lambda_{\varepsilon_\pi}(s_{t+1}) \quad (162)$$

Analytical Asset Correlation:

C.1 Stock and Bond Returns

The conditional covariance between nominal stock and bond returns can be expressed as follows:

$$Cov[r_{m,t+1}^{\$}(s_{t+1}), h_{t+1,n}^{\$(s_{t+1})}|I_t] = \quad (163)$$

$$E \left[E[(r_{m,t+1}^{\$(s_{t+1})} - E[r_{m,t+1}^{\$(s_{t+1})}|I_t])(h_{t+1,n}^{\$(s_{t+1})} - E[h_{t+1,n}^{\$(s_{t+1})}|I_t])|I_{t+1}]|I_t] \right]$$

Using the innovations derived in section B.2, the inner part of the above expression can be written as:

$$E[(r_{m,t+1}^{\$(s_{t+1})} - E[r_{m,t+1}^{\$(s_{t+1})}|I_t])(h_{t+1,n}^{\$(s_{t+1})} - E[h_{t+1,n}^{\$(s_{t+1})}|I_t])|I_{t+1}] = L(s_{t+1}) + R^{\$(s_t, s_{t+1})}H^{\$(s_t, s_{t+1})},$$

with $R^{\$(s_t, s_{t+1})}$ from Equation (144) and $H^{\$(s_t, s_{t+1})}$ from Equation ((147)), with D's replaced by $D^{\$}$'s and with

$$\begin{aligned} L(s_{t+1}) &= \sigma_c^2[k_{d,1}A_{d,1}(s_{t+1})\delta_1(s_{t+1}) + k_{d,1}A_{d,2}(s_{t+1})\delta_3(s_{t+1})][D_{1,n-1}^{\$(s_{t+1})}\delta_1(s_{t+1}) + D_{2,n-1}^{\$(s_{t+1})}\delta_3(s_{t+1})] \\ &+ \sigma_{\pi,t}^2[k_{d,1}A_{d,1}(s_{t+1})\delta_2(s_{t+1}) + k_{d,1}A_{d,2}(s_{t+1})\delta_4(s_{t+1})][D_{1,n-1}^{\$(s_{t+1})}\delta_2(s_{t+1}) + D_{2,n-1}^{\$(s_{t+1})}\delta_4(s_{t+1})] \\ &+ \sigma_v^2 k_{d,1}A_{d,3}(s_{t+1})D_{3,n-1}^{\$(s_{t+1})} \end{aligned} \quad (164)$$

Hence, the conditional covariance between stock and bond returns is:

$$Cov[r_{m,t+1}^{\$}(s_{t+1}), h_{t+1,n}^{\$}(s_{t+1})|I_t] = p_{s_t,0}[R^{\$}(s_t, 0)H^{\$}(s_t, 0) + L(0)] + p_{s_t,1}[R^{\$}(s_t, 1)H^{\$}(s_t, 1) + L(1)] \quad (165)$$

Equivalently the conditional covariance between stock and bond returns can also be written as:

$$Cov[r_{m,t+1}^{\$}(s_{t+1}), h_{t+1,n}^{\$}(s_{t+1})|I_t] = M_t + A + B\sigma_{\pi,t}^2 \quad (166)$$

$$\begin{aligned} B &= p_{s_t,0}[k_{d,1}A_{d,1}^0\delta_2^0 + k_{d,1}A_{d,2}^0\delta_4^0][D_{1,n-1}^{\$,0}\delta_2^0 + D_{2,n-1}^{\$,0}\delta_4^0] \\ &+ p_{s_t,1}[k_{d,1}A_{d,1}^1\delta_2^1 + k_{d,1}A_{d,2}^1\delta_4^1][D_{1,n-1}^{\$,1}\delta_2^1 + D_{2,n-1}^{\$,1}\delta_4^1] \end{aligned} \quad (167)$$

$$\begin{aligned} A &= p_{s_t,0} \left[\sigma_c^2[k_{d,1}A_{d,1}^0\delta_1^0 + k_{d,1}A_{d,2}^0\delta_3^0][D_{1,n-1}^{\$,0}\delta_1^0 + D_{2,n-1}^{\$,0}\delta_3^0] \right. \\ &\quad \left. + \sigma_v^2 k_{d,1}A_{d,3}^0 D_{3,n-1}^{\$,0} \right] \\ &+ p_{s_t,1} \left[\sigma_c^2[k_{d,1}A_{d,1}^1\delta_1^1 + k_{d,1}A_{d,2}^1\delta_3^1][D_{1,n-1}^{\$,1}\delta_1^1 + D_{2,n-1}^{\$,1}\delta_3^1] \right. \\ &\quad \left. + \sigma_v^2 k_{d,1}A_{d,3}^1 D_{3,n-1}^{\$,1} \right] \end{aligned} \quad (168)$$

$$M_t = p_{s_t,0}[R^{\$}(s_t, 0)H^{\$}(s_t, 0)] + p_{s_t,1}[R^{\$}(s_t, 1)H^{\$}(s_t, 1)] \quad (169)$$

C.2 Price-Dividend Ratio and Nominal Yields

Using the same approach as in section C.1, it is straightforward to show that the conditional covariance between price-dividend ratios and nominal yields can be stated as:

$$Cov[pd_{t+1}(s_{t+1}), y_{t+1,n}^{\$}(s_{t+1})|I_t] = M_t + A + B\sigma_{\pi,t}^2 \quad (170)$$

$$\begin{aligned} B = & -\frac{1}{n} \left[p_{s_t,0} [A_{d,1}^0 \delta_2^0 + A_{d,2}^0 \delta_4^0] [D_{1,n-1}^{\$,0} \delta_2^0 + D_{2,n-1}^{\$,0} \delta_4^0] \right. \\ & \left. + p_{s_t,1} [A_{d,1}^1 \delta_2^1 + A_{d,2}^1 \delta_4^1] [D_{1,n-1}^{\$,1} \delta_2^1 + D_{2,n-1}^{\$,1} \delta_4^1] \right] \end{aligned} \quad (171)$$

$$\begin{aligned} A = & -\frac{1}{n} \left[p_{s_t,0} \left[\sigma_c^2 [A_{d,1}^0 \delta_1^0 + A_{d,2}^0 \delta_3^0] [D_{1,n-1}^{\$,0} \delta_1^0 + D_{2,n-1}^{\$,0} \delta_3^0] \right. \right. \\ & \left. \left. + \sigma_v^2 A_{d,3}^0 D_{3,n-1}^{\$,0} \right] \right. \\ & \left. + p_{s_t,1} \left[\sigma_c^2 [A_{d,1}^1 \delta_1^1 + A_{d,2}^1 \delta_3^1] [D_{1,n-1}^{\$,1} \delta_1^1 + D_{2,n-1}^{\$,1} \delta_3^1] \right. \right. \\ & \left. \left. + \sigma_v^2 A_{d,3}^1 D_{3,n-1}^{\$,1} \right] \right] \end{aligned} \quad (172)$$

$$M_t = -\frac{1}{n} \left[p_{s_t,0} [S(s_t, 0)Q(s_t, 0)] + p_{s_t,1} [S(s_t, 1)Q(s_t, 1)] \right], \quad (173)$$

with $S(s_t, s_{t+1})$ given in Equation ((153)) and $Q(s_t, s_{t+1})$ given in Equation ((150)).

A.5 Additional Empirical Results

A.5.1 Regressions using conditional variance of inflation as measure for inflation risk

Table A5.1 Regressing Price-Dividend Ratios onto Inflation and Inflation Risk

	Data				
	β_π	t-stat	$\beta_{\sigma_\pi^2}$	t-stat	R^2
Full sample	-0.38	-5.18	-1.26	-1.99	0.31
Countercyclical state	-0.21	-3.73	-6.99	-5.33	0.62
Procyclical state	0.16	1.83	-1.17	-4.45	0.54

This table presents results from regressing log price-dividend ratios onto expected inflation (β_π) and inflation risk ($\beta_{\sigma_\pi^2}$): $pd_t = \alpha + \beta_\pi x_{\pi,t} + \beta_{\sigma_\pi^2} \sigma_{\pi,t}^2 + \epsilon_t$. Inflation risk is measured as the conditional variance of inflation and is estimated from an AR(1)-GARCH(1,1) on expected inflation. Inflation expectations are created by projecting quarterly demeaned inflation onto lagged growth, inflation, and yield spread. All regressions are run contemporaneously. Standard errors are computed using Newey-West (1987) with 4 lags. The countercyclical state refers to 1965:1-1999:4 and the procyclical state to 2000:1-2011:4.

Table A5.2 Regressing Nominal Yields onto Inflation and Inflation Risk

	Data				
	β_π	t-stat	$\beta_{\sigma_\pi^2}$	t-stat	R^2
Full sample	2.67	5.34	-5.10	-1.22	0.26
Countercyclical state	0.63	1.22	55.76	4.03	0.48
Procyclical state	0.70	2.80	-6.44	-2.85	0.38

This table presents results from regressing 5-year nominal interest rates onto expected inflation (β_π) and inflation risk ($\beta_{\sigma_\pi^2}$): $y_{t,5y}^S = \alpha + \beta_\pi x_{\pi,t} + \beta_{\sigma_\pi^2} \sigma_{\pi,t}^2 + \epsilon_t$. Inflation risk is measured as the conditional variance of inflation and is estimated from an AR(1)-GARCH(1,1) on expected inflation. Inflation expectations are created by projecting quarterly demeaned inflation onto lagged growth, inflation, and yield spread. All regressions are run contemporaneously. Standard errors are computed using Newey-West (1987) with 4 lags. The countercyclical state refers to 1965:1-1999:4 and the procyclical state to 2000:1-2011:4.

Table A5.3 Predicting Covariance of Stock and Bond Returns

	Data		
	$\beta_{\sigma_\pi^2}$	t-stat	R^2
Full sample	-0.53	-3.26	0.04
Countercyclical state	1.06	2.81	0.12
Procyclical state	-0.20	-0.96	0.00

This table presents results from predicting quarterly covariances between returns on US stocks and Treasury bonds using inflation risk: $\sigma(r_{stock,t+1}, r_{bond,t+1}) = \alpha + \beta_{\sigma_\pi^2} \sigma_\pi^2 + \epsilon_t$. Dependent variable is the realized quarterly covariance between stock and bond returns computed using daily returns. The independent variable consists of inflation volatility estimated from an AR(1)-GARCH(1,1) on expected inflation. Standard errors are computed using Newey-West (1987) with 4 lags. The countercyclical state refers to 1968:4-1999:4 and the procyclical state to 2000:1-2011:4.

A.5.2 Regressions using dispersion of CPI forecasts as measure for inflation risk

Table A5.5 Regressing Price-Dividend Ratios onto Inflation and Inflation Risk

	Data				
	β_π	t-stat	$\beta_{\sigma_\pi^2}$	t-stat	R^2
Full sample	-0.13	-0.97	-26.29	-4.55	0.31
Countercyclical state	-0.49	-3.59	-14.79	-3.15	0.40
Procyclical state	0.12	1.85	-45.26	-4.19	0.51

This table presents results from regressing log price-dividend ratios onto expected inflation (β_π) and inflation risk ($\beta_{\sigma_\pi^2}$): $pd_t = \alpha + \beta_\pi x_{\pi,t} + \beta_{\sigma_\pi^2} \sigma_\pi^2 + \epsilon_t$. Inflation risk is measured as the cross-sectional variance of individual forecasters of CPI inflation, taken from Survey of Professional Forecasters. Inflation expectations are created by projecting quarterly demeaned inflation onto lagged growth, inflation, and yield spread. All regressions are run contemporaneously. Standard errors are computed using Newey-West (1987) with 4 lags. The countercyclical state refers to 1968:4-1999:4 and the procyclical state to 2000:1-2011:4.

Table A5.4 Regressing Nominal Yields onto Inflation and Inflation Risk

	Data				
	β_π	t-stat	$\beta_{\sigma_\pi^2}$	t-stat	R^2
Full sample	3.39	4.96	148.95	4.11	0.33
Countercyclical state	3.70	3.88	108.32	3.27	0.45
Procyclical state	0.74	1.89	-156.12	-2.46	0.20

This table presents results from regressing 5-year nominal interest rates onto expected inflation (β_π) and inflation risk ($\beta_{\sigma_\pi^2}$): $y_{t,5y}^s = \alpha + \beta_\pi x_{\pi,t} + \beta_{\sigma_\pi^2} \sigma_{\pi,t}^2 + \epsilon_t$. Inflation risk is measured as the cross-sectional variance of individual forecasters of CPI inflation, taken from Survey of Professional Forecasters. Inflation expectations are created by projecting quarterly demeaned inflation onto lagged growth, inflation, and yield spread. All regressions are run contemporaneously. Standard errors are computed using Newey-West (1987) with 4 lags. The countercyclical state refers to 1968:4-1999:4 and the procyclical state to 2000:1-2011:4.

Table A5.6 Predicting Covariance of Stock and Bond Returns

	Data		
	$\beta_{\sigma_\pi^2}$	t-stat	R^2
Full sample	4.52	1.93	0.02
Countercyclical state	3.80	4.04	0.18
Procyclical state	-29.99	-2.34	0.13

This table presents results from predicting quarterly covariances between returns on US stocks and Treasury bonds using inflation risk: $\sigma(r_{stock,t+1}, r_{bond,t+1}) = \alpha + \beta_{\sigma_\pi^2} \sigma_{\pi,t}^2 + \epsilon_t$. Dependent variable is the realized quarterly covariance between stock and bond returns computed using daily returns. Independent variable is the cross-sectional variance of individual forecasters of CPI inflation, taken from Survey of Professional Forecasters. Standard errors are computed using Newey-West (1987) with 4 lags. The countercyclical state refers to 1968:4-1999:4 and the procyclical state to 2000:1-2011:4.

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Table 1: **Unconditional Correlation Matrices and Variances over Subperiods**

	Correlation Matrix				Variance
	Cg	Inf	Stock	Bond	
<i>1965-2011</i>					
Cg	1.000	-0.161	0.191	-0.128	0.227
Inf		1.000	-0.086	-0.241	0.415
Stock			1.000	0.073	72.54
Bond				1.000	12.12
<i>1965-2000</i>					
Cg	1.000	-0.420	0.154	-0.083	0.212
Inf		1.000	-0.201	-0.202	0.412
Stock			1.000	0.283	66.70
Bond				1.000	12.77
<i>2001-2011</i>					
Cg	1.000	0.298	0.349	-0.177	0.144
Inf		1.000	0.273	-0.329	0.263
Stock			1.000	-0.631	94.70
Bond				1.000	9.583

This table presents unconditional correlation matrices and variances based on quarterly data over the full sample period 1965-2011 and the considered subperiods 1965-2000 (countercyclical period) and 2001-2011 (procyclical period). Stock and bond returns are excess returns.

Table 2: **Jennrich (1970) Test of Equality of Correlation Matrices over Subperiods**

Model	Degree of freedom	1965-2000 compared to 2001-2011	
		Statistics	p-value
Cg-Inf-Stock-Bond	6	50.344	0.000
Cg-Inf	1	19.368	0.000
Stock-Bond	1	27.577	0.000
Stock-Inflation	1	7.446	0.006

This table presents a formal test for constant unconditional correlation: The Jennrich (1970) test of equality of two correlation matrices computed over independent subsamples. The Jennrich test statistic is asymptotically distributed as a Chi-square with the degree of freedom equal to the number of correlation coefficients.

Table 3: **Regressing Price-Dividend Ratios onto Inflation and Inflation Risk**

	Data				
	β_π	t-stat	$\beta_{\sigma_\pi^2}$	t-stat	R^2
Full sample	-0.16	-2.22	-47.76	-5.10	0.48
Countercyclical state	-0.20	-3.34	-30.20	-3.86	0.44
Procyclical state	0.10	1.09	-84.58	-6.35	0.51

This table presents results from regressing log price-dividend ratios onto expected inflation (β_π) and inflation risk ($\beta_{\sigma_\pi^2}$): $pd_t = \alpha + \beta_\pi x_{\pi,t} + \beta_{\sigma_\pi^2} \sigma_{\pi,t}^2 + \epsilon_t$. Inflation risk is measured as the cross-sectional variance of individual forecasters of the GDP price deflator (PGDP), taken from Survey of Professional Forecasters. Inflation expectations are created by projecting quarterly demeaned inflation onto lagged growth, inflation, and yield spread. All regressions are run contemporaneously. Standard errors are computed using Newey-West (1987) with 4 lags. The countercyclical state refers to 1965:1-1999:4 and the procyclical state to 2000:1-2011:4.

Table 4: **Regressing Nominal Yields onto Inflation and Inflation Risk**

	Data				
	β_π	t-stat	$\beta_{\sigma_\pi^2}$	t-stat	R^2
Full sample	1.61	2.86	244.68	3.19	0.34
Countercyclical state	0.42	0.78	223.89	3.04	0.25
Procyclical state	0.50	1.40	-382.72	-2.49	0.27

This table presents results from regressing 5-year nominal interest rates onto expected inflation (β_π) and inflation risk ($\beta_{\sigma_\pi^2}$): $y_{t,5y}^s = \alpha + \beta_\pi x_{\pi,t} + \beta_{\sigma_\pi^2} \sigma_{\pi,t}^2 + \epsilon_t$. Inflation risk is measured as the cross-sectional variance of individual forecasters of the GDP price deflator (PGDP), taken from Survey of Professional Forecasters. Inflation expectations are created by projecting quarterly demeaned inflation onto lagged growth, inflation, and yield spread. All regressions are run contemporaneously. Standard errors are computed using Newey-West (1987) with 4 lags. The countercyclical state refers to 1965:1-1999:4 and the procyclical state to 2000:1-2011:4

Table 5: **Predicting Covariance of Stock and Bond Returns**

	Data		
	$\beta_{\sigma_\pi^2}$	t-stat	R^2
Full sample	11.33	4.02	0.08
Countercyclical state	5.16	2.30	0.08
Procyclical state	-40.64	-2.33	0.06

This table presents results from predicting quarterly covariances between returns on US stocks and Treasury bonds using inflation risk: $\sigma(r_{stock,t+1}, r_{bond,t+1}) = \alpha + \beta_{\sigma_\pi^2} \sigma_{\pi,t}^2 + \epsilon_t$. Dependent variable is the realized quarterly covariance between stock and bond returns computed using daily returns. Independent variable is the cross-sectional variance of individual forecasters of the GDP price deflator (PGDP), taken from Survey of Professional Forecasters. Standard errors are computed using Newey-West (1987) with 4 lags. The countercyclical state refers to 1965:1-1999:4 and the procyclical state to 2000:1-2011:4.

Table 6: **Estimation Results for Markov-Switching Model**

	Countercyclical State				Procyclical State			
	Cg	Inf	Stock	Bond	Cg	Inf	Stock	Bond
<i>β Matrix</i>								
Cg	0.347 (0.084)	-0.122 (0.062)	0.012 (0.005)	0.015 (0.011)	0.675 (0.087)	0.033 (0.067)	0.006 (0.004)	0.006 (0.011)
Inf	0.138 (0.067)	0.866 (0.050)	-0.004 (0.004)	-0.008 (0.009)	0.667 (0.170)	-0.164 (0.127)	0.007 (0.007)	-0.047 (0.020)
Stock	-1.274 (1.627)	-1.251 (1.221)	0.001 (0.051)	0.315 (0.197)	4.812 (3.983)	-2.826 (2.812)	0.150 (0.205)	0.388 (0.6163)
Bond	-0.426 (0.714)	-1.167 (0.532)	-0.060 (0.038)	-0.047 (0.089)	-2.605 (1.228)	2.181 (0.950)	-0.070 (0.056)	-0.068 (0.177)
<i>Ω Matrix</i>								
Cg	0.174 (0.021)	-0.040 (0.012)	0.671 (0.277)	-0.182 (0.124)	0.036 (0.009)	0.007 (0.011)	0.277 (0.291)	0.013 (0.067)
Inf		0.110 (0.013)	-0.614 (0.218)	-0.202 (0.098)		0.122 (0.026)	1.127 (0.556)	-0.257 (0.155)
Stock			59.012 (7.106)	8.865 (2.417)			99.156 (21.473)	-16.286 (4.940)
Bond				12.248 (1.466)				7.762 (1.658)
<i>μ Vector</i>								
μ	0.638 (0.123)	0.060 (0.099)	3.268 (2.468)	2.101 (1.056)	0.139 (0.057)	0.468 (0.104)	-0.042 (0.162)	0.722 (0.697)
<i>Probs</i>								
p_{11}	0.977 (0.014)							
p_{22}	0.939 (0.039)							
fval	1259.7							

This table presents results from estimating a two-regime MS-VAR model using maximum likelihood (see Hamilton (1989,1994)). Sample period is 1965:1 to 2011:4 and the model is formulated as indicated in Equation (1). Stock and bond returns are excess returns. Standard errors are computed using the Hessian. Tomorrow's state s_{t+1} is presumed to follow a two-state Markov chain with transition probabilities $p_{ij} = P(s_{t+1} = j | s_t = i)$ and where $\sum_{j=1}^N p_{ij} = 1$ and $0 < p_{ij} < 1$. The probability of ending up in tomorrow's state $s_{t+1} = (0, 1)$ given today's state $s_t = (0, 1)$ is governed by the transitional probability matrix:

$$P = \begin{bmatrix} p_{00} & p_{10} \\ p_{01} & p_{11} \end{bmatrix}.$$

Table 7: **Unconditional Correlations implied by Estimated Markov-Switching Model**

	Countercyclical State				Procyclical State			
	Cg	Inf	Stock	Bond	Cg	Inf	Stock	Bond
Unconditional Correlations								
Cg	1	-0.42	0.20	-0.11	1	0.41	0.18	-0.11
Inf		1	-0.23	-0.20		1	0.30	-0.32
Stock			1	0.32			1	-0.60
Bond				1				1

This table presents the unconditional correlation matrix implied by the estimated empirical Markov-Switching model.

Table 8: **Jennrich (1970) Test of Equality of Correlation Matrices implied by Estimated Markov-Switching Model**

Model	Degree of freedom	1965-2000 compared to 2001-2011	
		Statistics	p-value
Cg-Inf-Stock-Bond	6	58.993	0.000
Cg-Inf	1	24.755	0.000
Stock-Bond	1	28.671	0.000
Stock-Inflation	1	9.350	0.002

This table presents a formal test for constant unconditional correlation implied by the Markov-Switching model across regimes: The Jennrich (1979) test of equality of two correlation matrices computed over independent subsamples. The Jennrich test statistic is asymptotically distributed as a Chi-square with the degree of freedom equal to the number of correlation coefficients.

Table 9: **LR test statistic for regime-independent correlations**

Model	Degree of freedom	LR test statistic $H_0: \rho(S_t) = \rho$	
		Statistics	p-value
Cg-Inf-Stock-Bond	6	25.387	0.0003

This table presents a LR test for the null hypothesis of a constant conditional correlation matrix across regimes. The LR test statistic is $2(\ln L(\theta) - \ln L(\theta_0))$, with θ_0 corresponding to the parameter vector resulting under the null hypothesis.

Table 10: **Calibrated Model Parameters**

	Countercyclical state	Procyclical state
μ_c	0.0081	0.0039
μ_π	0.0114	0.0067
μ_d	0.0035	0.0035
β_1	0.951	0.995
β_2	-0.012	0.012
β_4	0.90	0.40
δ_1	0.12	0.12
δ_2	-0.15	0.2
δ_3	-0.1	0.9
δ_4	0.9	0.9
σ_π	0.0035	0.0035
v_π	0.98	0.98
$\sigma_\nu * 10^{-6}$	1	1
σ_c	0.0033	0.0033
ϕ	2	2
φ	5	5
γ	10	10
ψ	2	2
δ	0.998	0.998
p_{00}	0.98	
p_{11}	0.94	

This table presents calibrated parameters for the two economic states. Parameters are calibrated as to match both standard macro and asset pricing moments as well as the various relations between stocks and bonds and between inflation and asset prices. The transition probabilities are the ones we estimated in the empirical RS model. The countercyclical state refers to 1965:1-2001:2 and the procyclical state to 2001:3-2011:4.

Table 11: **Macro Moments**

	Countercyclical State			Procyclical State			Full Sample		
	Model	Data	SE	Model	Data	SE	Model	Data	SE
<u>Consumption growth, Δc</u>									
Mean	0.81	0.81	(0.06)	0.38	0.39	(0.11)	0.70	0.71	(0.06)
Std.Dev.	0.46	0.46	(0.04)	0.54	0.38	(0.09)	0.51	0.48	(0.04)
AC(1)	0.46	0.39	(0.05)	0.62	0.77	(0.12)	0.57	0.52	(0.08)
<u>Inflation, π</u>									
Mean	1.14	1.14	(0.11)	0.67	0.67	(0.08)	1.03	1.04	(0.09)
Std.Dev.	0.71	0.64	(0.08)	0.50	0.51	(0.13)	0.70	0.65	(0.08)
AC(1)	0.90	0.84	(0.05)	0.41	0.27	(0.14)	0.84	0.77	(0.07)
<u>Dividend growth, Δd</u>									
Mean	0.35	0.22	(0.18)	0.32	0.82	(0.75)	0.34	0.35	(0.22)
Std.Dev.	1.76	1.46	(0.15)	1.86	2.82	(0.66)	1.79	1.86	(0.24)
AC(4)	0.11	-0.02	(0.13)	0.20	0.06	(0.17)	0.14	0.07	(0.12)

This table presents unconditional quarterly moments of the macro variables. Sample statistics refer to the countercyclical period 1965:1-2001:2, to the procyclical period 2001:3-2011:4 and to the full sample period 1965:1-2011:4. Model statistics are based on a simulation of 150000 quarters. AC(k) denotes the autocorrelation for k lags. Standard errors for the observed data, denoted SE, are computed as in Newey West (1987), using four lags.

Table 12: Asset Price Moments

	Countercyclical State			Procyclical State			Full Sample		
	Model	Data	SE	Model	Data	SE	Model	Data	SE
<u>Equity</u>									
$E(r_m - r_f)$	0.81	1.05	(0.62)	0.88	0.45	(1.54)	0.82	0.91	(0.60)
$\sigma(r_m - r_f)$	3.69	8.12	(0.75)	4.58	9.73	(1.40)	3.93	8.52	(0.67)
$E(pd)$	3.40	3.44	(0.06)	3.37	3.83	(0.05)	3.39	3.52	(0.06)
$\sigma(pd)$	0.13	0.34	(0.04)	0.21	0.19	(0.04)	0.15	0.35	(0.03)
<u>Nominal Bonds</u>									
$E(y_{3m}^{\$})$	6.26	6.47	(0.44)	3.26	1.78	(0.54)	5.52	5.43	(0.48)
$E(y_{5y}^{\$} - y_{3m}^{\$})$	0.44	0.98	(0.18)	0.49	1.40	(0.27)	0.45	1.07	(0.15)
$\sigma(y_{3m}^{\$})$	2.51	2.57	(0.40)	2.30	1.68	(0.27)	2.78	3.09	(0.38)
$\sigma(y_{5y}^{\$} - y_{3m}^{\$})$	1.69	1.16	(0.12)	1.72	0.94	(0.13)	1.70	1.13	(0.10)
<u>Real Bonds</u>									
$E(y_{3m})$	1.73			0.49			1.43		
$E(y_{5y})$	1.26			0.49			1.07		
$\sigma(y_{3m})$	0.65			0.91			0.90		
$\sigma(y_{5y})$	0.52			0.80			0.69		

This table presents unconditional asset-price moments. All results are reported on an annualized basis. Sample statistics refer to the countercyclical period 1965:1-2001:2, to the procyclical period 2001:3-2011:4 and to the full sample period 1965:1-2011:4. Model statistics are based on a simulation of 150000 quarters. Standard errors for the observed data, denoted SE, are computed as in Newey West (1987), using four lags.

Table 13: Macro and Asset Correlations

	Countercyclical State			Procyclical State			Full Sample		
	Model	Data	SE	Model	Data	SE	Model	Data	SE
$Corr(\Delta c, \pi)$	-0.42	-0.42	(0.10)	0.15	0.30	(0.19)	-0.16	-0.16	(0.15)
$Corr(dp, y_{5y}^{\$})$	0.45	0.68	(0.05)	-0.97	-0.65	(0.06)	-0.09	0.70	(0.06)
$Corr(r_{stock}, r_{bond})$	0.43	0.28	(0.08)	-0.50	-0.63	(0.08)	0.10	0.07	(0.11)
$Corr(\Delta c, \Delta d)$	0.25	0.19	(0.07)	0.36	0.49	(0.07)	0.26	0.19	(0.09)
$Corr(\Delta d, \pi)$	-0.21	-0.13	(0.08)	0.09	0.06	(0.06)	-0.14	-0.10	(0.08)

This table presents unconditional correlations of macro and asset-price data. $Corr(\Delta c, \pi)$ refers to the correlation between consumption growth and inflation, $Corr(dp, y_{5y}^{\$})$ refers to the correlation between dividend yields and nominal yields, $Corr(r_{stock}, r_{bond})$ refers to the correlation between stock and bond returns, $Corr(\Delta c, \Delta d)$ is the correlation between consumption growth and dividend growth and $Corr(\Delta d, \pi)$ refers to the correlation between dividend growth and inflation. Model statistics are based on a simulation of 150000 quarters. The countercyclical state refers to 1965:1-2001:2 and the procyclical state to 2001:3-2011:4. Standard errors for the observed data, denoted SE, are computed as in Newey West (1987), using four lags.

Table 14: Inflation and Asset Prices - Model Regressions

	Price-Dividend Ratios			Nominal Interest Rates			Stock-Bond Covariance
	β_π	$\beta_{\sigma_\pi^2}$	R^2	β_π	$\beta_{\sigma_\pi^2}$	R^2	$\beta_{\sigma_\pi^2}$
Full Sample	-0.09	-1.23	0.30	0.10	0.60	0.22	
Countercyclical state	-0.12	-1.22	0.64	0.12	2.91	0.86	39.78
Procyclical state	0.11	-1.25	0.14	0.86	-5.75	0.32	-31.81

This table presents results from running regressions using simulated asset prices and inflation inside the model. The first regression regresses log price-dividend ratios onto expected inflation and the conditional inflation variance, i.e. the two state variables: $pd_t = \alpha + \beta_\pi x_{\pi,t} + \beta_{\sigma_\pi^2} \sigma_\pi^2 + \epsilon_t$. The second regression regresses the nominal 5-year interest rate onto expected inflation and the conditional inflation variance: $y_{t,5y}^{\$} = \alpha + \beta_\pi x_{\pi,t} + \beta_{\sigma_\pi^2} \sigma_\pi^2 + \epsilon_t$. Finally, we report the analytical coefficient governing the relation between the conditional stock-bond covariance and the conditional inflation variance from Equation (22). We report the analytical coefficient since the model does not allow for simulation of realized covariances based on daily returns, as in data. Model statistics are based on a simulation of 150000 quarters.

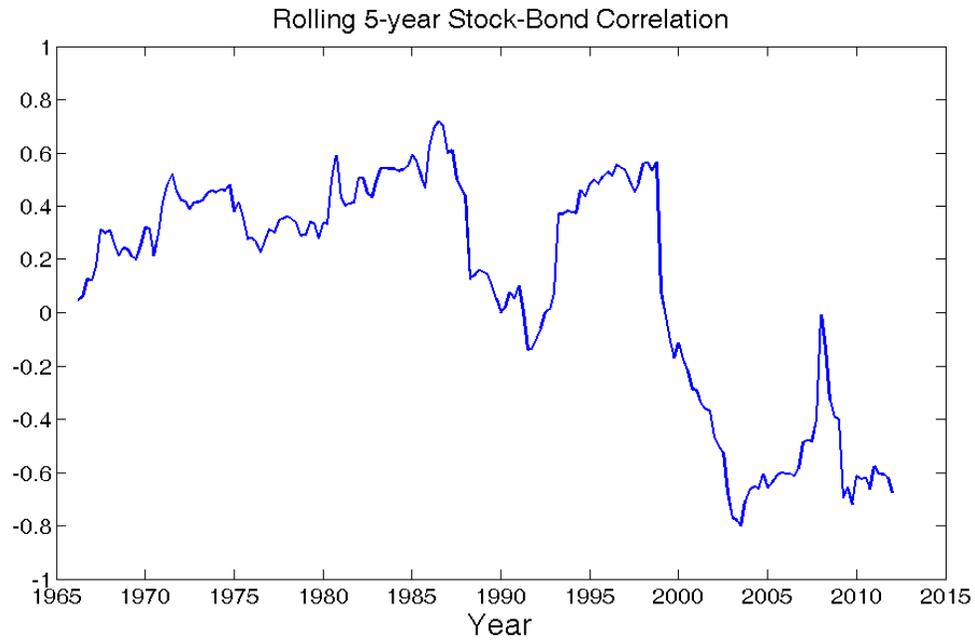


Figure 1: Rolling 5-year correlations between returns on US stocks and nominal 5-year Treasury bonds. Correlations are based on quarterly excess returns and cover the period Mar 1961-Dec 2011.

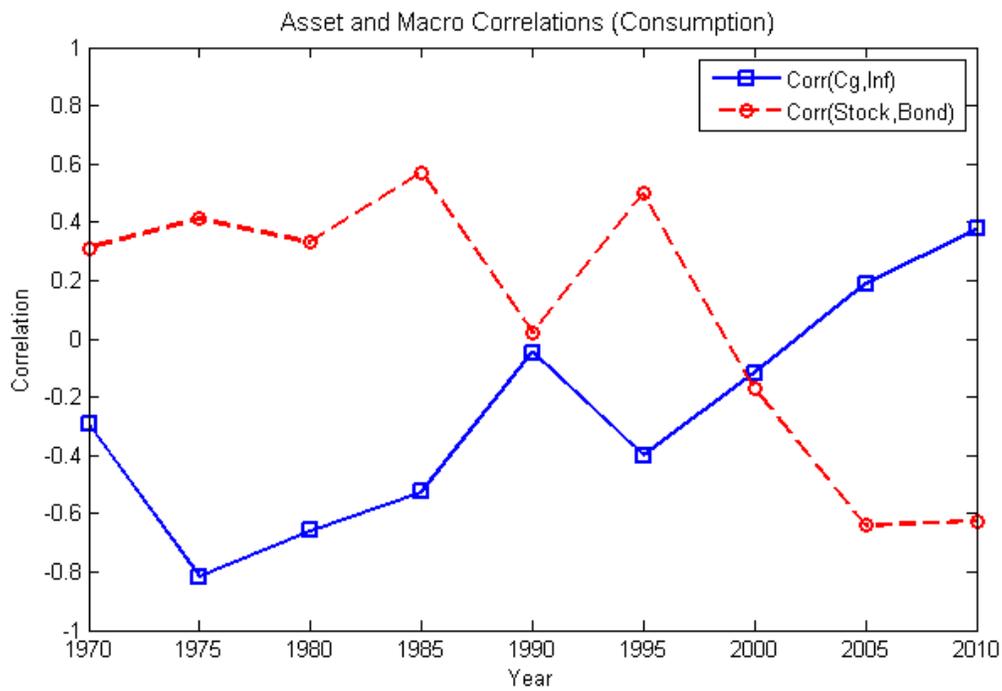


Figure 2: Correlation between quarterly real consumption growth and inflation and between the returns on US stocks and long-term Treasury bonds. Correlations are computed for non-overlapping 5-year intervals over the period 1965:1-2011:4.

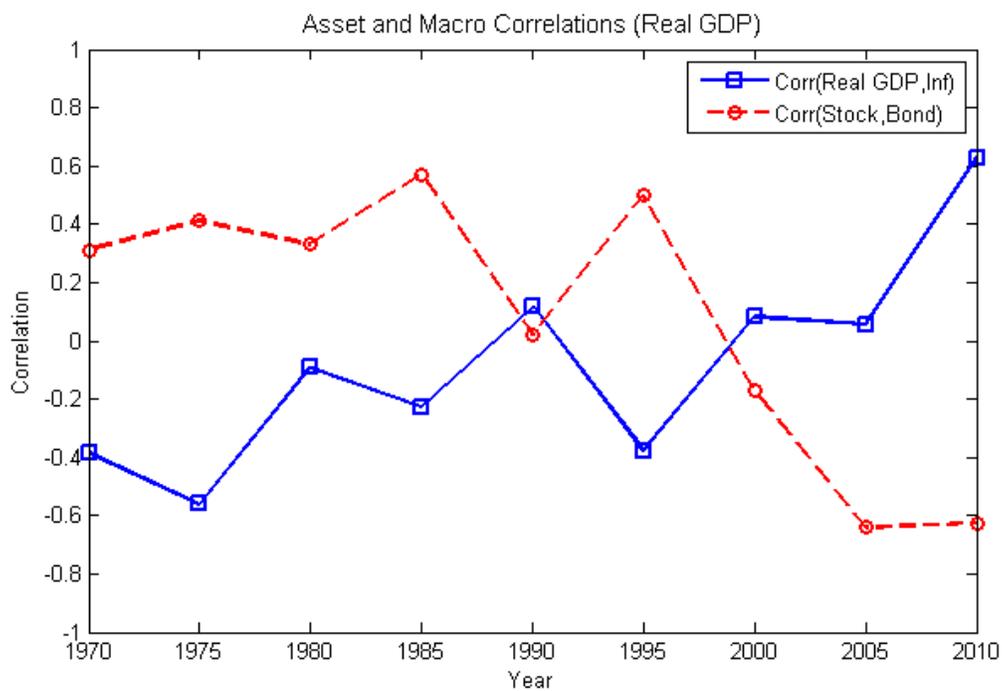


Figure 3: Correlation between quarterly real GDP growth and inflation and between the returns on US stocks and long-term Treasury bonds. Correlations are computed for non-overlapping 5-year intervals over the period 1965:1-2011:4.

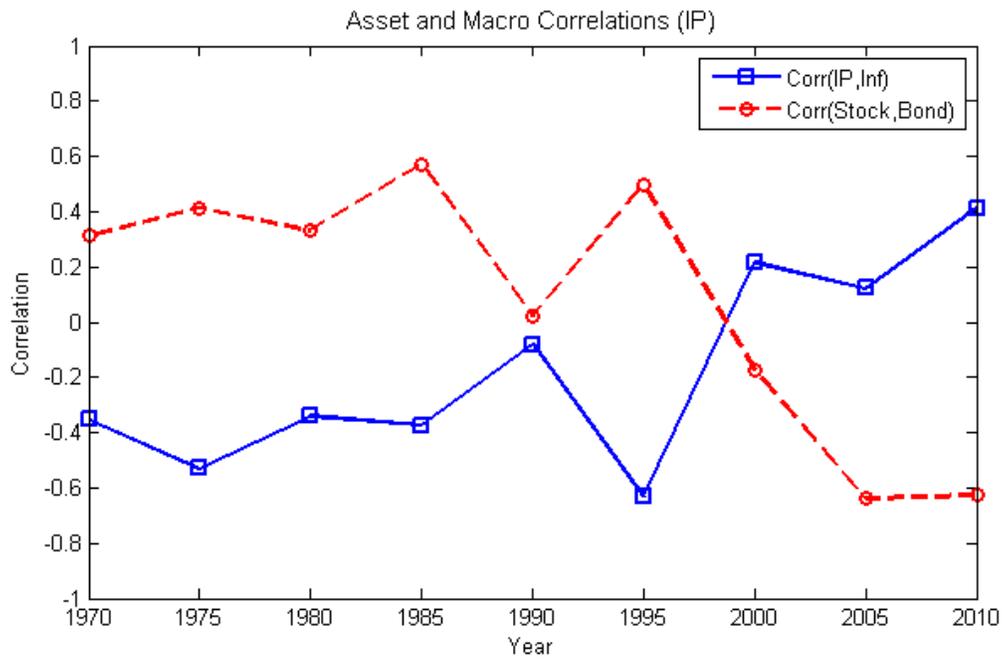


Figure 4: Correlation between quarterly US industrial production growth and inflation and between the returns on US stocks and long-term Treasury bonds with maturities. Correlations are computed for non-overlapping 5-year intervals over the period 1965:1-2011:4.

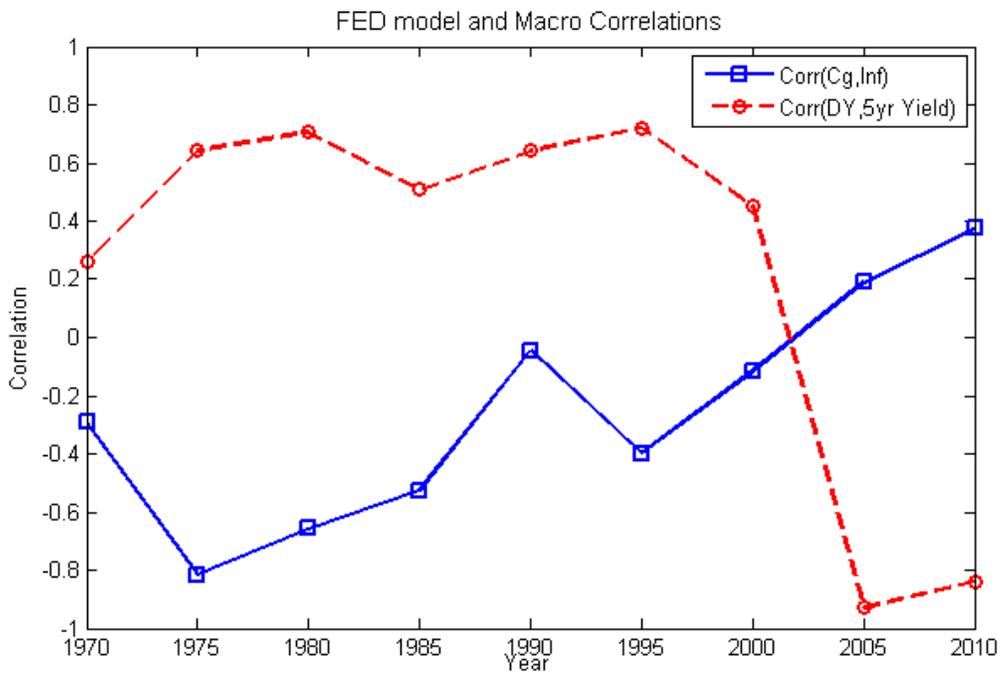


Figure 5: Correlation between quarterly real consumption growth and inflation and between the dividend yield on US stocks and 5-year nominal yields on US Treasury bonds. Correlations are computed for non-overlapping 5-year intervals over the period 1965:1-2011:4.

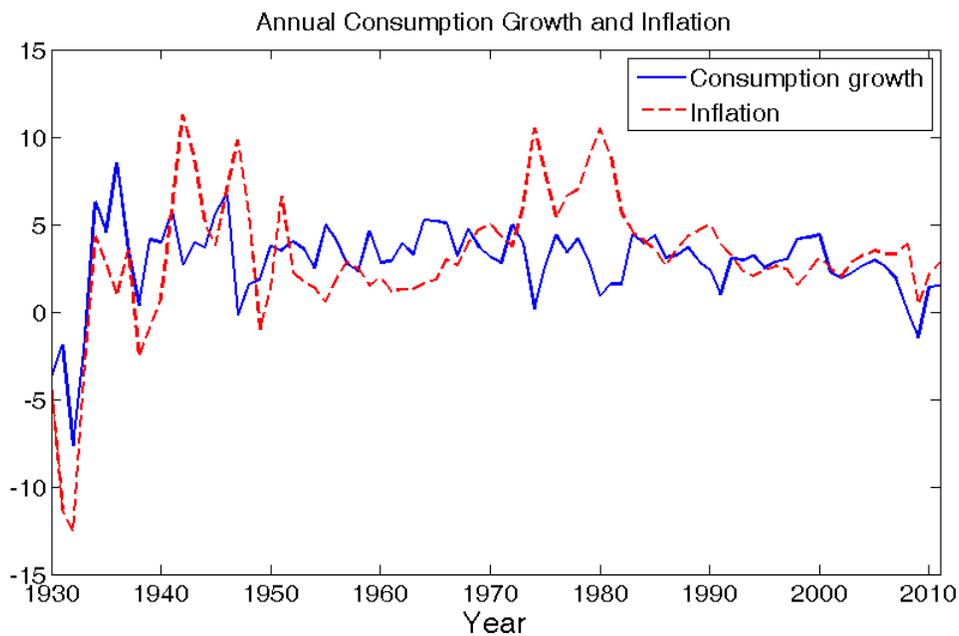


Figure 6: Annual real consumption growth and inflation over the period 1930-2011.

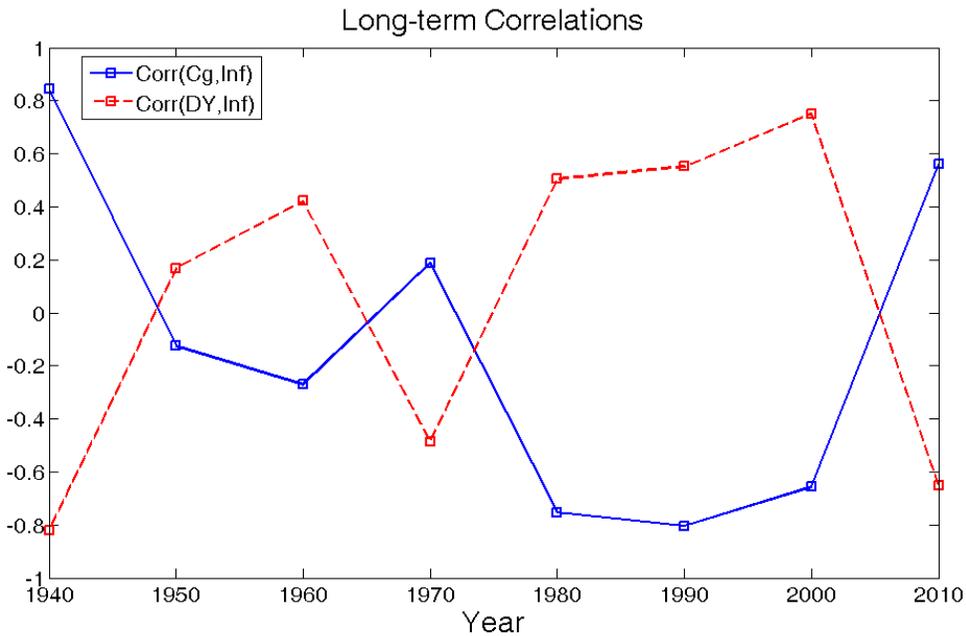


Figure 7: Correlations between annual real consumption growth and inflation and between dividend yields and inflation. Correlations are computed for non-overlapping 10-year intervals over the period 1930-2011.

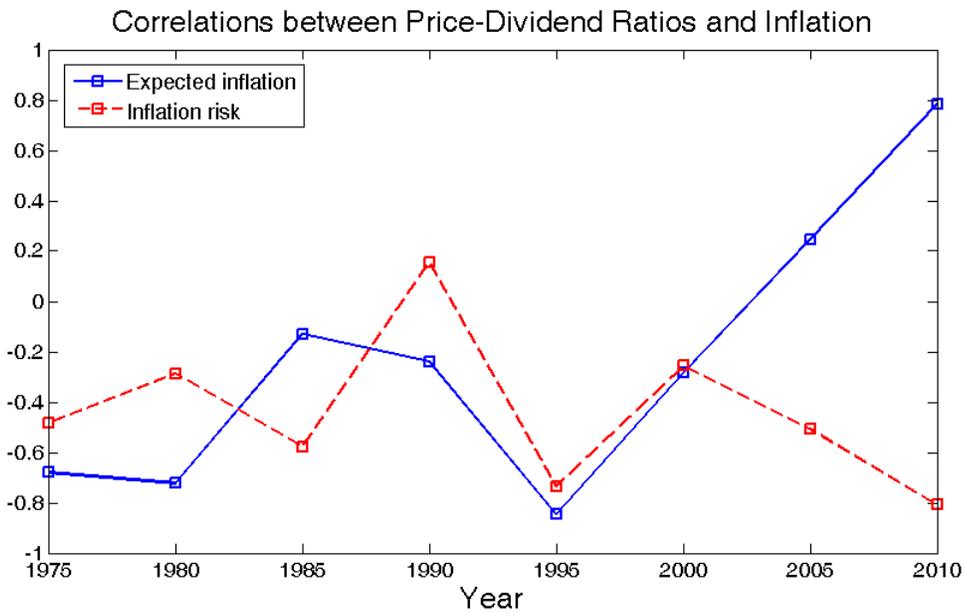


Figure 8: Correlations between log price-dividend ratios and expected inflation and inflation risk. Correlations are based on quarterly data and computed for non-overlapping 5-year intervals over the period 1970-2011. Inflation expectations are created by projecting quarterly inflation onto lagged growth, inflation, and yield spread. Inflation risk is measured as dispersion of inflation forecasts (PGDP) taken from Survey of Professional Forecasters.

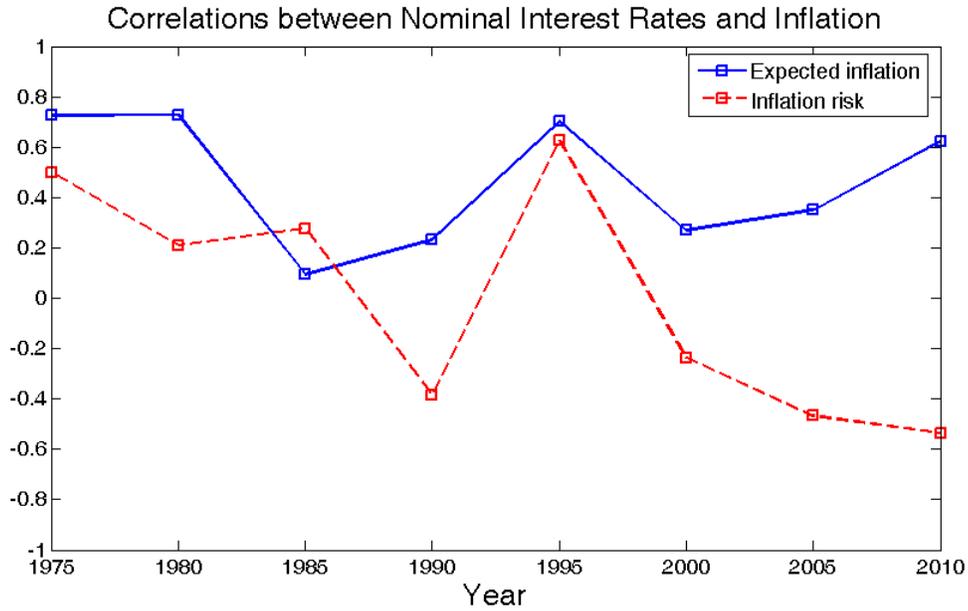


Figure 9: Correlations between the 5-year nominal interest rate and expected inflation and inflation risk. Correlations are based on quarterly data and computed for non-overlapping 5-year intervals over the period 1970-2011. Inflation expectations are created by projecting quarterly inflation onto lagged growth, inflation, and yield spread. Inflation risk is measured as dispersion of inflation forecasts (PGDP) taken from Survey of Professional Forecasters.

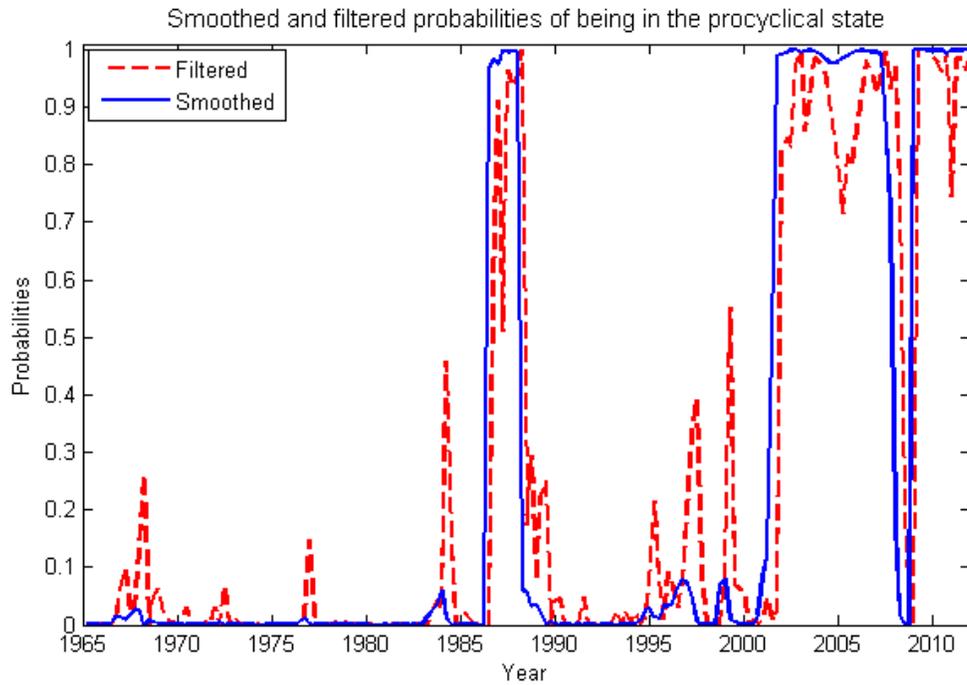


Figure 10: Filtered and smoothed probabilities of being in the procyclical inflation state. Probabilities are based on the estimated Markov-switching model described in Equation (1).

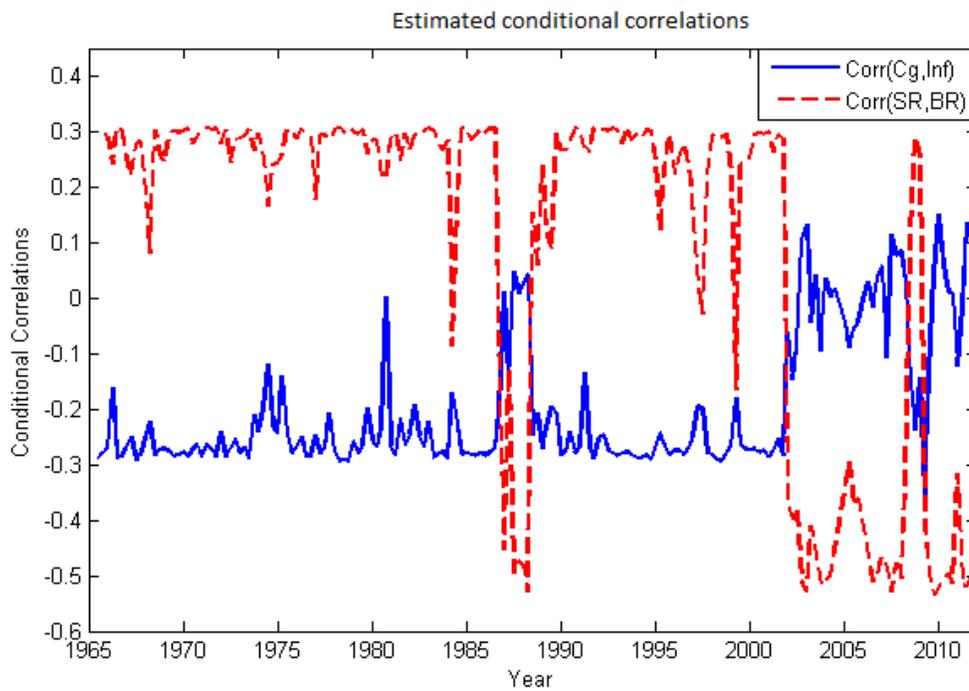


Figure 11: Empirically estimated quarterly conditional correlation between consumption growth and inflation and between stock and bond returns based on the estimated Markov-switching model described in Equation (1).

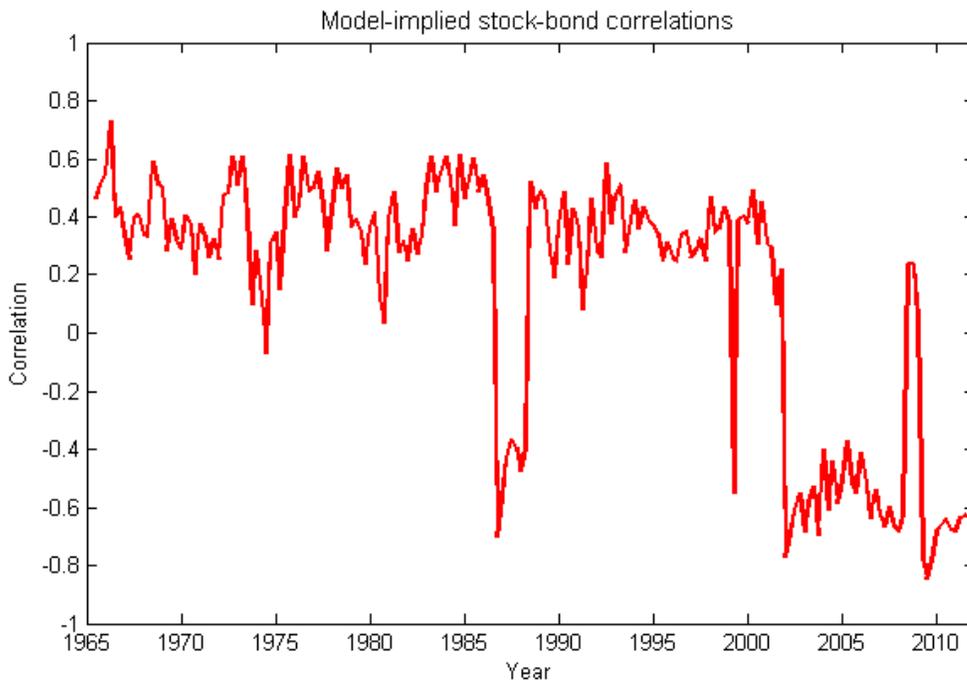


Figure 12: Model-implied quarterly conditional correlation between stock and 5-yr bond returns. Empirical proxies for the state variables, expected inflation and inflation volatility, are used to compute correlations. Inflation expectations are created by projecting quarterly demeaned inflation onto lagged growth, inflation, and yield spread. Inflation risk is measured as conditional inflation volatility based on an AR(1)-GARCH(1,1) on expected inflation. Correlations are computed using Equation (22) for the covariances together with analytical expressions for the volatility of stock and bond returns.